FUNCTION

EXERCISE – I HINTS & SOLUTIONS

Sol.1 **D**

$$f(x) = \frac{\sqrt{-\log_{0.3}(x-1)}}{\sqrt{x^2+2x+8}}$$

$x - 1 > 0 \quad \Rightarrow \quad x > 1$

$\&\ x^2 + 2x - 8 > 0 \ \Rightarrow\ D < 0 \quad \Rightarrow\ x \in R$

$\&\ -\log_{0.3}(x-1) \geq 0 \quad \Rightarrow\ \log_{0.3}(x-1) \leq 0$

$\Rightarrow\ x - 1 \geq 1\ \Rightarrow\ x \geq 2\ \ \therefore$ Domain $x \in [2, \infty)$

Sol.2 **D**

$$f(x) = \log_{1/2}\left(-\log_2\left(1+\frac{1}{4\sqrt{x}}\right)-1\right)$$

$$\Rightarrow\ -\log_2\left(1+\frac{1}{x^{1/4}}\right) - 1 > 0$$

$$\Rightarrow\ -\log_2\left(1+\frac{1}{x^{1/4}}\right) > 1\ ; \quad x > 0$$

$$\Rightarrow\ \log_2\left(1+\frac{1}{x^{1/4}}\right) < -1\ \ \Rightarrow 1 + \frac{1}{x^{1/4}} < \frac{1}{2}$$

$$\Rightarrow\ \frac{1}{x^{1/4}} < -\frac{1}{2}\ \ \Rightarrow\ x \in \phi$$

Sol.3 **B**

$q^2 - 4pr = 0,\ p > 0$

$f(x) = \log\,(px^3 + (p+q)\,x^2 + (q+r)\,x + r)$

$\Rightarrow\ px^3 + (p+q)\,x^2 + (q+r)\,x + r > 0$

$\Rightarrow\ (px^3 + px^2) + (qx^2 + qx) + (rx + r) > 0$

$\Rightarrow\ px^2\,(x+1) + qx\,(x+1) + r(x+1) > 0$

$\Rightarrow\ (x+1)\,(px^2 + qx + r) > 0 \quad \Rightarrow\ D = q^2 - 4pr$

Means it is perfect square

$$x = -\frac{b}{2a}$$

$$(x+1)\left(x+\frac{q}{2p}\right)^2 > 0 \Rightarrow x + 1 > 0 \Rightarrow x > -1\ \left(x \neq \frac{-q}{2p}\right)$$

$$\Rightarrow\ x \in (-1, \infty) - \left\{-\frac{q}{2p}\right\}$$

$$\therefore\quad x \in R - \left[(-\infty,-1) \cup \left(-\frac{q}{2p}\right)\right]$$

Sol.4 **A**

Domains of $f(x)$ is $(-\infty, 0]$

Domains of $f\,(6\{x\}^2 - 5\,\{x\} + 1)$

$$6\{x\}^2 - 5\{x\} + 1 \leq 0 \qquad \Rightarrow \qquad \frac{1}{3} \leq \{x\} \leq \frac{1}{2}$$

$$\Rightarrow n + \frac{1}{3} \leq x \leq n + \frac{1}{2}\ ;\ n \in I\ \Rightarrow \bigcup_{n\in I}\left[n+\frac{1}{3}, n+\frac{1}{2}\right]$$

Sol.5 **B**

$$f(x) = \sqrt{-\log_{\frac{x+4}{2}}\left(\log_2\frac{2x-1}{3+x}\right)}$$

Case-1 : $\dfrac{x+4}{2} > 1\ \Rightarrow\ x > -2$

$$-\log_{\frac{x+4}{2}}\log_2\left(\frac{2x-1}{x+3}\right) \geq 0$$

$$\Rightarrow\ \log_{\frac{x+4}{2}}\log_2\left(\frac{2x-1}{x+3}\right) \leq 0$$

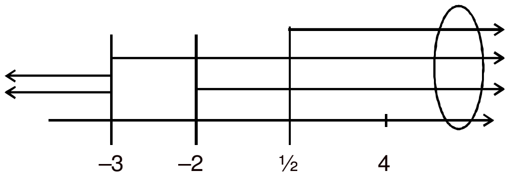
$$\Rightarrow\ \log_2\left(\frac{2x-1}{x+3}\right) \leq 1 \quad \Rightarrow \quad \frac{2x-1}{x+3} \leq 2$$

$$\Rightarrow\ \frac{7}{x+3} \geq 0 \qquad \Rightarrow \qquad x > -3$$

$$\&\ \frac{2x-1}{x+3} > 0 \qquad \Rightarrow \qquad x > \frac{1}{2}\ ; x < -3$$

$$\&\ \log_2\left(\frac{2x-1}{x+3}\right) > 0 \quad \Rightarrow \quad \frac{2x-1}{x+3} > 1$$

$$\Rightarrow\ \frac{x-4}{x+3} > 0 \qquad \Rightarrow \qquad x > 4\ ;\ x < -3$$



$\therefore\quad x \in (4, \infty)$ ....(i)

Case-2 : $0 < \dfrac{x+4}{2} < 1 \quad \Rightarrow \quad -4 < x < -2$

$$-\log_{\frac{x+4}{2}}\log_2\frac{2x-1}{x+3} \geq 0$$

$\Rightarrow \log_{\frac{x+4}{2}} \log_2 \dfrac{2x-1}{x+3} \le 0 \Rightarrow \log_2 \dfrac{2x-1}{x+3} \ge 1$

$\Rightarrow \dfrac{7}{x+3} \le 0 \qquad \Rightarrow \qquad x < -3$

$\& \quad \dfrac{2x-1}{x+3} > 0 \qquad \Rightarrow \qquad x > \dfrac{1}{2}\ ;\ x < -3$

$\& \quad \log_2\left(\dfrac{2x-1}{x+3}\right) > 0 \Rightarrow \qquad \dfrac{x-4}{x+3} > 0$

$\Rightarrow \quad x > 4\ ;\ x < -3 \ \therefore\ x \in (-4, -3) \quad ...(2)$

$(1) \cup (2) \quad \Rightarrow \qquad x \in (-4, -3) \cup (4, \infty)$

Sol.6 A

$\sqrt{\log_{1/3}\log_4([x]^2-5)}$

$\Rightarrow \log_{1/3}\log_4([x]^2-5) \ge 0 \Rightarrow 0 < \log_4([x]^2-5) \le 1$

$\Rightarrow \quad 1 < [x]^2 - 5 \le 4 \qquad \Rightarrow \qquad 6 < [x]^2 \le 9$

[x] always gives integer value so square of GTF will also give Integer value. In between 6 and 9 are only perfect square value possible.

$[x]^2 = 9 \Rightarrow [x] = 3 \qquad [x] = -3$

$3 \le x < 4 \qquad -3 \le x < -2$

$\therefore \quad x \in [-3, -2) \cup [3, 4)$

Sol.7 D

$f(x) = (x^{12} - x^9 + x^4 - x + 1)^{-1/2}$

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

If $x < 0$ +ve –(–ve) +ve –(–ve) +ve > 0

If $x = 0$ $f(x) > 0$

If $x > 0$ $x^9(x^3-1) + x(x^3-1) + 1$

$\downarrow \qquad \downarrow$

Now if $x \ge 1$ +ve +ve + 1 > 0

If $0 < x < 1$

$1 - x + x^4 - x^9 + x^{12}$

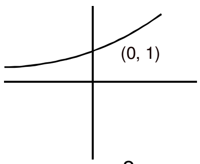
$(1-x) + x^4(1-x^5) + x^{12} > 0 \quad \Rightarrow \quad x \in R$

Sol.8 B

$f(x) = 4^x + 2^x + 1$

Let $2^x = t\ ;\ t \in (0, \infty)$

$= t^2 + t + 1$



$f(x) = \left(t+\dfrac{1}{2}\right)^2 - \dfrac{1}{4} + 1 = \left(t+\dfrac{1}{2}\right)^2 + \dfrac{3}{4} = (1, \infty)$

Sol.9 B

$f(x) = \log_{\sqrt{5}}\{\sqrt{2}(\sin x - \cos x) + 3\}$

$\Rightarrow \qquad -\sqrt{2} \le \sin x - \cos \le \sqrt{2}$

$\Rightarrow \qquad -2 \le \sqrt{2}(\sin x - \cos x) \le 2$

$\Rightarrow 1 \le \sqrt{2}(\sin x - \cos x) + 3 \le 5$

$\Rightarrow 0 \le \log_{\sqrt{5}}[m] \le 2 \qquad \therefore$ Range $\in [0, 2]$

Sol.10 D

$f(x) = \log_{\sqrt{2}}(2 - \log_2(16\sin^2 x + 1))$

$\Rightarrow \quad 0 \le \sin^2 x \le 1 \qquad \Rightarrow \quad 0 \le 16\sin^2 x \le 16$

$\Rightarrow \quad 1 \le (16\sin^2 - x + 1) \le 17$

$\Rightarrow \quad 0 \le \log_2(16\sin^2 x + 1) \le \log_2 17$

$\Rightarrow \quad -\log_2 17 \le -\log_2(16\sin^2 x + 1) \le 0$

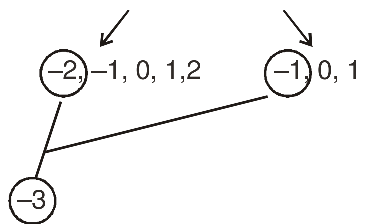
$\Rightarrow \quad 2 - \log_2 17 \le 2 - \log_2(16\sin^2 x + 1) \le 2$

$\Rightarrow \quad \log_{\sqrt{2}}(2 - \log_2 17) \le \log_{\sqrt{2}} M \le 2$

$\therefore \quad -\infty < \log_{\sqrt{2}} M \le 2$

Sol.11 D

$[2\cos x] + [\sin x] = -3$



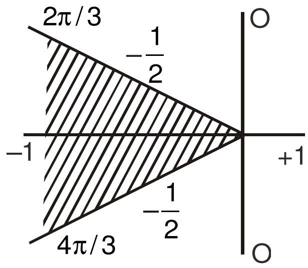
$-2 \le 2\cos x < -1$

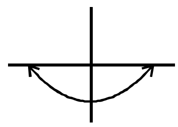
$-1 \le \cos x < -\dfrac{1}{2}$

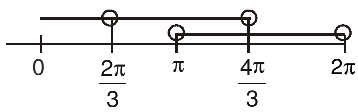
$x \in \left(\dfrac{2\pi}{3}, \dfrac{4\pi}{3}\right)$

$-1 \le \sin x < 0$

$x \in (\pi, 2\pi)$







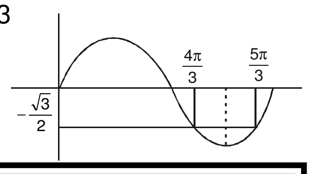
$\therefore \quad \pi < x < \dfrac{4\pi}{3}$

$f(x) = \sin x + \sqrt{3}\cos x$

$= 2\left[\dfrac{1}{2}\sin x + \dfrac{\sqrt{3}}{2}\cos x\right] = 2\sin\left(x + \dfrac{\pi}{3}\right) = 2\sin\theta$

$\pi + \dfrac{\pi}{3} < \theta < \dfrac{\pi}{3} + \dfrac{4\pi}{3}$

$\Rightarrow \dfrac{4\pi}{3} < \theta < \dfrac{5\pi}{3}$



$\Rightarrow \quad -1 \le \sin\theta < -\dfrac{\sqrt{3}}{2}$

$\Rightarrow \quad -2 \le 2\sin\theta < -\sqrt{3} \quad \therefore [-2, -\sqrt{3})$

Sol.12 A

$f(x) = {}^{7-x}P_{x-3}$ ${}^{n}P_r$ $n \in N$

$7 - x > 0 \Rightarrow x < 7$ $r \in w$

$x - 3 \ge 0 \Rightarrow x \ge 3$ $n \ge r$

$7 - x \ge x - 3 \Rightarrow x \le 5$

$x \in \{3, 4, 5\}$ $x = 3 \quad {}^{4}P_0 = 1$

$x = 4 \quad {}^{3}P_1 = 3$

$x = 5 \quad {}^{2}P_2 = 2$

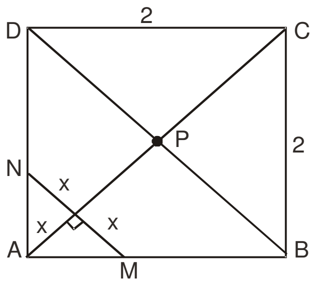
Sol.13 C

$$f(x) = \begin{vmatrix} \cos\frac{x}{2} & 1 & 1 \\ 1 & \cos\frac{x}{2} & -\cos\frac{x}{2} \\ -\cos\frac{x}{2} & 1 & -1 \end{vmatrix} = 2 + 2\cos^2\frac{x}{2}$$

$0 \le \cos^2\dfrac{x}{2} \le 1 \quad \Rightarrow \quad 0 \le 2\cos^2\dfrac{x}{2} \le 2$

$\Rightarrow 2 \le 2 + 2\cos^2\dfrac{x}{2} \le 4 \quad \therefore \quad 2 \le y \le 4$

Sol.14 B



Area (ΔAMN)

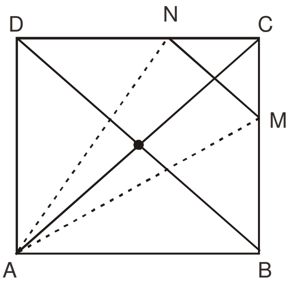
$= \dfrac{1}{2}(2x)\,x = x^2$

$(AP = \sqrt{2})$

$0 < x \le \sqrt{2}$

$0 < x^2 \le 2$

$\sqrt{2} \le x < 2\sqrt{2}$



$PC = 2\sqrt{2} - x$

$MN = 2(2\sqrt{2} - x)$

Area (ΔAMN)

$= \dfrac{1}{2}\,2(2\sqrt{2} - x)\,x$

$= 2\sqrt{2}\,x - x^2 = -(x^2 - 2\sqrt{2}\,x)$

$= -[(x - \sqrt{2})^2 - 2] = 2 - (x - \sqrt{2})^2$

$\Rightarrow \quad x = \sqrt{2},\ y = 2\ ;\ x = 2\sqrt{2},\ y = 0 \ \therefore\ y \in (0, 2]$

Sol.15 D

$$f(x) = \frac{e^x - e^{-|x|}}{e^x + e^{|x|}} = \begin{cases} \dfrac{e^x - e^{-x}}{2e^x} & ;\ x \ge 0 \\ 0 & ;\ x < 0 \end{cases}$$

for $x \ge 0$

$y = \dfrac{e^x - e^{-x}}{2e^x} = \dfrac{e^{2x} - 1}{2e^{2x}}$

Let $t = e^x \ \Rightarrow\ y = \dfrac{t^2 - 1}{2t^2}$

$e^x \ge 1 \quad \Rightarrow \quad t \ge 1$

$\therefore \quad 2yt^2 = t^2 - 1$

$t^2 \ge 1 \ \Rightarrow\ t^2 = \dfrac{1}{1 - 2y} \ge 1 \ \Rightarrow\ 0 \le y < \dfrac{1}{2}$

Sol.16 C

$f(x) = \dfrac{\sin^2 x + 4\sin x + 5}{2\sin^2 x + 8\sin x + 8}$

$f(x) = \dfrac{1}{2}\left[\dfrac{2\sin^2 x + 8\sin x + 10}{2\sin^2 x + 8\sin x + 8}\right]$

$= \dfrac{1}{2}\left[1 + \dfrac{2}{2\sin^2 x + 8\sin x + 8}\right]$

$= \dfrac{1}{2}\left[1 + \dfrac{1}{(\sin x + 2)^2}\right]$

$f(x)|_{max.} = \dfrac{1}{2}[1 + 1] = 1$

$f(x)|_{min.} = \dfrac{1}{2}\left[1 + \dfrac{1}{9}\right] = \dfrac{5}{9} \qquad \therefore \text{Range} \in \left[\dfrac{5}{9}, 1\right]$

Sol.17 C

$[x] + 2\{-x\} = 3x$

Case–1 : $x \in I$; $x + 0 = 3x \Rightarrow 2x = 0 \Rightarrow x = 0$

Case–2 : $x \notin I$

$\Rightarrow [x] + 2(1 - \{x\}) = 3x \Rightarrow [x] + 2 - 2(x - [x]) = 3x$

$\Rightarrow [x] + 2 - 2x + 2[x] = 3x \ \Rightarrow 3[x] = 5x - 2$ ...(1)

& $x - \{x\} + 2 - 2\{x\} = 3x$

$\Rightarrow \quad 2 - 3\{x\} = 3x \qquad \Rightarrow \quad 0 \le \{x\} < 1$

$\Rightarrow \quad 0 \le 3\{x\} < 3 \qquad \Rightarrow \quad 0 \le 2 - 2x < 3$

$\Rightarrow \quad -2 \le -2x < 1 \qquad \Rightarrow \quad -1 < 2x \le 2$

$\therefore \quad -\dfrac{1}{2} < x \le 1$

If $-\frac{1}{2} < x < 0$

from (1) $\Rightarrow -3 = 5x - 2 \quad \Rightarrow x = -\frac{1}{5}$

If $0 \le x < 1$

from (1) $\Rightarrow 0 = 5x - 2 \Rightarrow x = 2/5 \quad \therefore$ 3 solution

If x = 1 (reject as $x \in I$ is already taken)

Sol.18 B

$[\sin^{-1} x] = x - [x]$

$[\sin^{-1} x] = \{x\}$

$-2, -1, 0, 1 \qquad 0 \le \{x\} < 1$

possible only if $[\sin^{-1} x] = 0 \quad \& \quad \{x\} = 0$

$0 \le \sin^{-1} x < 1 \quad ; \quad x = 0$

$0 \le x < \sin 1$

common solution is x = 0

Sol.19 A

$$\left[\frac{1}{2} + \frac{1999}{2000}\right] = [0.5 + 0.995] = 1$$

$$\vdots$$

$$\left[\frac{1}{2} + \frac{1000}{2000}\right] = [0.5 + 0.5] = 1$$

$(1 + 1 + ... + 1)_{1000 \text{ times}} = 1000$

Sol.20 A

$f(x) = \text{sgn}\,[x + 1]$

$= 1 \quad$ if $[x + 1] > 0$

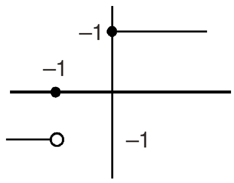
$\Rightarrow [x] > -1 \quad \therefore \quad x \ge 0$

$= 0 \quad$ if $[x + 1] = 0$

$\Rightarrow [x] = -1 \;\therefore\; -1 \le x < 0$

$= -1 \quad$ if $[x + 1] < 0$

$\Rightarrow [x] < -1 \quad \therefore \quad x < -1$



Sol.21 B

$f(x) = 2 \sin^2\theta + 4 \cos (x + \theta) \sin x . \sin\theta + \cos (2x + 2\theta)$

$f(x) = \cos 2x$

$$\Rightarrow \quad f\left(\frac{\pi}{4} - x\right) = \cos\left(2\left(\frac{\pi}{4} - x\right)\right) = \sin 2x$$

$$\Rightarrow \quad f^2(x) + f^2\left(\frac{\pi}{4} - x\right) = 1$$

Sol.22 C

$0 < A < 1, 0 < B < 1, 0 < c < 1$

$\Rightarrow \quad 0 < A + B + C < 3$

$p = [A + B + C] = 2, \; q = [A] + [B] + [C] = 0$

Maximum value of p – q means maximum value of p and minimum value of q = 2 – 0 = 2

Sol.23 B

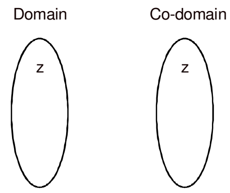
$z \to$ integer

$f(x) = ax^2 + bx + c\, f(1)$

$= a + b + c = I$

$f(1) - f(2) = a + b \in I$

$f(0) = c \in I$



Sol.24 C

(A) $e^{(\ell n x)/2}$ and $\sqrt{x} \Rightarrow D_1 \in (0, \infty)$; $D_2 \in [0, \infty)$

Domain are not same so not identical

(B) $\tan^{-1} (\tan x)$ and $\cot^{-1} (\cot x)$

Domain are not same so non identical

(C) $\cos^2x + \sin^{-1}x$ and $\sin^2x + \cos^4x$

$\Rightarrow x \in R \;\&\; x \in R$ Identical

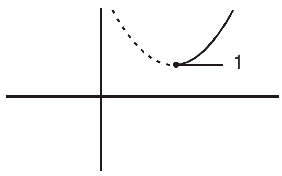
Sol.25 B

$f : [2, \infty) \to y$

$f(x) = x^2 - 4x + 5$

one – one and onto

$$\frac{-D}{4a} = -\frac{(16 - 20)}{4} = 1$$



for one-one and onto y = 1 $\quad \therefore \quad$ range $[1, \infty)$

Sol.26 D

$$f(x) = \frac{2x^2 - x + 5}{7x^2 + 2x + 10} = \frac{g(x)}{h(x)}$$

Quadratic expression g(x) & h(x) are always positive so f(x) is always positive.

so range ≠ co-domain.

Sol.27 A

$f : R \to R \quad f(x) = x^3 + x^2 + 3x + \sin x$

$$f'(x) = \underbrace{3x^2 + 2x + 3}_{1} + \underbrace{\cos x}_{(-1,1)} > 0 \;\forall x \in R$$

$$\frac{-D}{4a} = -\frac{(4 - 36)}{12} = \frac{32}{12} = 2.57$$

f(x) = one-one, $\quad$ Range = Co-domain

Sol.28 A

$$f(x) = \frac{4a - 7}{3} x^3 + (a - 3)^{x^2} + x + 5$$

Case – 1 : $a = \frac{7}{4}$

$f(x) = -\frac{5}{4} x^2 + x + 5 \qquad$ which can't be one-one

Case – 2 : $a \neq \frac{7}{4}$

$f'(x) = (4a - 7)x^2 + 2(a - 3)x + 1$

$D \leq 0$

$\Rightarrow \quad 4(a-3)^2 - 4(4a-7) \leq 0$

$\Rightarrow \quad a^2 - 6a + 9 - 4a + 7 \leq 0$

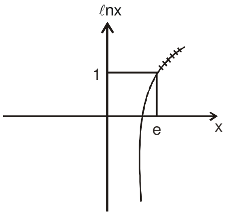
$\Rightarrow \quad a^2 - 10a + 16 \leq 0$

$\Rightarrow \quad (a-8)(a-2) \leq 0 \quad \therefore\ 2 \leq a \leq 8$

Sol.29 C

$f : (e, \infty) \to R$

$f(x) = \ell n(\ell n(\ell n\, x))$



$e < x < \infty$

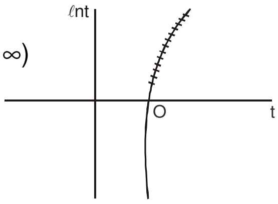
Let $\ell n\, x = t$

$t \in (1, \infty) \Rightarrow \ell n\, t \in (0, \infty)$

$1 < \ell nx < \infty$

$0 < \ell n(\ell n\, x) < \infty$

$-\infty < \ell n(\ell n(\ell n\, x)) < \infty$



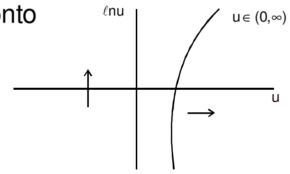
Range = Co-domain onto

Let $\ell n\, t = u$

Let $\ell nx = t$; $t \in (1, \infty)$

$\ell n\, t \in (0, \infty)$

one-one

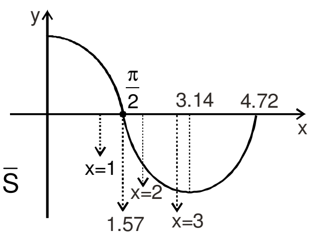


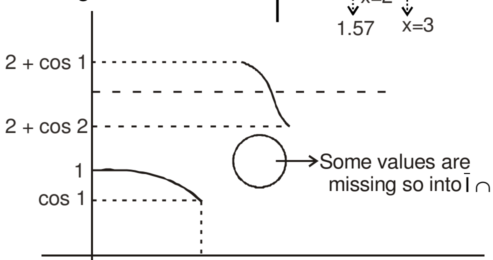
Sol.30 C

$f(x) = 2[x] + \cos x$

$f : R \to R$

Some values are missing so into $\bar{I} \cap \bar{S}$





$0 \leq x < 1 \quad f(x) = \cos x$

$1 \leq x < 2 \quad f(x) = 2 + \cos x$

$2 \leq x < 3 \quad f(x) = 4 + \cos x$

$3 \leq x < 4 \to$ curve will turns thats why many-one.

Sol.31 D

$f : R \to S$; $f(x) = \sin x - \sqrt{3}\cos x + 1$

$\Rightarrow \quad -2 \leq \sin x - \sqrt{3}\cos x \leq 2$

$\Rightarrow \quad -1 \leq \sin x - \sqrt{3}\cos x + 1 \leq 3$

Range $\in [-1, 3]$

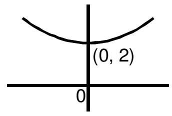
Sol.32 D

$f : R \to R$; $f(x) = 6^x + 6^{|x|}$

$x \geq 0 \quad f(x) = 2.6^x$

$x < 0 \quad f(x) = 6^x + 6^{-x} = 6^x + \frac{1}{6^x} \geq 2$

Many-one into



Sol.33 D

$f(x) = px + \sin x$

$f'(x) = p + \cos x$

$p \neq 0$ for converging ranges of $f(x)$ is $(-\infty, \infty)$

$f'(x) = P + \cos x > 0$ or $< 0 \quad \forall x \in R$

$P \in (-\infty, -1) \cup (1, \infty) \to f'(x)$ will not be zero.

Sol.34 B

$f : S \to R^+$, $f(\Delta)$ = area of the Δ

$S \to$ set of triangle ; $R^+ \to$ set of real values

for one base there are many triangle can possible. So many-one.

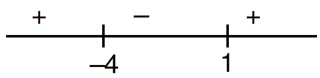
Sol.35 C

$f : R \to R$; $f(x) = \frac{x^2 - 4}{x^2 + 1} \Rightarrow f(-x) = \frac{x^2 - 4}{x^2 + 1} = f(x)$

$f(x)$ is even that's why many-one.

$y = \frac{x^2 - 4}{x^2 + 1} \Rightarrow yx^2 + y = x^2 - 4$

$\Rightarrow x^2 = \frac{y + 4}{1 - y} \geq 0$



$\Rightarrow \frac{y + 4}{y - 1} \leq 0 \qquad \therefore\ y \in [-4, 1)$

Range $\neq$ Co-Domain $\Rightarrow$ into

Sol.36 B

$f : (-\infty, 1) \to [0, e^5]$; $f(x) = e^{-(x^2 - 3x + 2)} = e^{-x^2 + 3x - 2}$

$t(x) = -x^2 + 3x - 2$

$= -(x-2)(x-1)$

$-\infty < t(x) < 0$

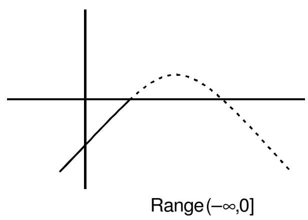
$0 < e^{t(x)} < 1$

$f(x) = e^{t(x)}$

$f(x_1) = f(x_2)$

$e^{t(x_1)} = e^{t(x_2)} \Rightarrow t(x_1) = t(x_2)$

$x_1 = x_2 \Rightarrow t$ is one-one $\Rightarrow f$ is one-one function.



Sol.37 C

$f(g(x)) = \cot^{-1}(2x - x^2)$

$-\infty < 2x - x^2 \le 1$

But domain of f(x) is R^+

$0 < 2x - x^2 \le 1 \;\Rightarrow\; \frac{\pi}{2} > \cot^{-1}(2x - x^2) \ge \frac{\pi}{4}$

Range $\in \left[\frac{\pi}{4}, \frac{\pi}{2}\right)$

Sol.38 B

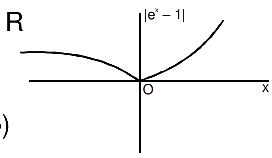
$f(x) = |x - 1| : f : R^+ \to R$

$g(x) = e^x,\ g : [-1, \infty) \to R$

$f[g(x)] = f(e^x) = |e^x - 1|$

Range $\in (0, \infty)$

Domain = g(x) = $[-1, \infty)$



Sol.39 B

$g(x) = 1 + x - [x]$ $\quad f(x) = \begin{cases} -1 & x < 0 \\ 0 & x = 0 \\ 1 & x > 0 \end{cases}$

$\Rightarrow f(x) = \text{sgn}x$

$f[g(x)] = f(1 + x - [x]) = \text{sgn}(1 + x - [x])$

$= \text{sgn}\underbrace{(1 + \{x\})}_{\text{positive}} = 1$

Sol.40 A

$f : [0, 1] \to [1, 2] \quad g : [1, 2] \to [0, 1]$

$f(x) = 1 + x \qquad g(x) = 2 - x$

$gof(x) = g[f(x)] = g(1 + x) = 2 - (1 + x) = 1 - x$

Linear polynomial thats why.

Sol.41 D

$$f(x) = \frac{x + |x|}{2} = \begin{cases} x^2 & ; x \ge 0 \\ x & ; x \ge 0 \\ 0 & ; x < 0 \end{cases} ;\ g(x) = x \quad x < 0$$

$$gof = g[f(x)] = \begin{cases} g(x) & ; x \ge 0 \\ x^2 & ; x \ge 0 \\ g(0) & ; x \le 0 \\ 0 & ; x \le 0 \end{cases}$$

$$fog(x) = f[g(x)] = \begin{cases} f(x^2) & ; x \ge 0 \\ x^2 & ; x \ge 0 \\ f(x) & ; x < 0 \\ 0 & ; x < 0 \end{cases}$$

$\Rightarrow \quad fog(x) = gof(x).$

Sol.42 D

$y = f(x)$

$f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2} \quad (a \ne 0)$

$f\left(x + \frac{1}{x}\right) = \left(x + \frac{1}{x}\right)^2 - 2 \qquad \Rightarrow f(x) = x^2 - 2$

Sol.43 C

$f(1) = 1$

$f(n + 1) = 2 f(n) + 1$

$f(2) = f(1 + 1) = 2 + 1$

$f(3) = 2f(2) + 1 = 2(2 + 1) + 1 = 4 + 2 + 1$

$f(4) = 2f(3) + 1 = 2(4 + 2 + 1) + 1 = 8 + 4 + 2 + 1$

$f(n) = 2^{n-1} + 2^{n-2} + \ldots\ldots + 8 + 4 + 2 + 1$

$= 1 + 2 + 4 + 8 + \ldots + 2^{n-1} = \frac{2^n - 1}{2 - 1} = 2^n - 1$

Sol.44 B

$x^2 f(x) + f(1 - x) = 2x - x^4$

replace $x \to 1 - x$

$(1 - x)^2 \cdot f(1 - x) + f(x) = 2(1 - x) - (1 - x)^4$

$f(x) = 1 - x^2$

Sol.45 D

$f(x - y) = f(x) f(y) - f(a - x) f(a + y) \quad (\therefore f(0) = 1)$

put y = 0

$f(x) = f(x) f(0) - f(a - x) f(a) \Rightarrow f(a - x) f(a) = 0 \ldots(1)$

put x = a, y = a – x

$\Rightarrow \quad f(x) = f(a - x) f(a) - f(0).f(2a - x)$

$\Rightarrow \quad f(x) = 0 - f(2a - x) \;\Rightarrow\; f(2a - x) = -f(x)$

Sol.46 D

$f : R \to R$

$f(x + y) = f(x) + f(y)$

$f(1) = 7$

$f(2) = f(1) + f(1) = 14$

$f(3) = f(2) + f(1) = 14 + 7 = 21$

$f(4) = f(2) + f(1) = 21 + 7 = 28$

$f(5) = f(4) + f(1) = 28 + 7 = 35$

$\Rightarrow \quad f(1) + f(3) + f(3) + f(4) + f(5) + \ldots\ldots$

$= 7 + 14 + 21 + 28 + 35 + \ldots\ldots..$

$= 7(1 + 2 + 3 + 4 + 5 + \ldots\ldots...) = \frac{7n(n+1)}{2}$

Sol.47 B

$$f(x) = \log\left(\frac{1+\sin x}{1-\sin x}\right)$$

$$f(-x) = \log\left(\frac{1-\sin x}{1+\sin x}\right)$$

$$\Rightarrow \quad f(x) + f(-x) = \log\left(\frac{1+\sin x}{1-\sin x}\times\frac{1-\sin x}{1+\sin x}\right)$$

$$\Rightarrow \quad f(x) + f(-x) = 0 \qquad \therefore\ f(-x) = -f(x) \quad \text{odd}$$

Sol.48 D

$$f(x) = \frac{a^x - 1}{x^n(a^x+1)}$$

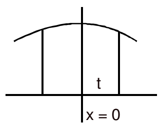
$$f(-x) = \frac{a^{-x}-1}{(-x)^n(a^{-x}+1)} = \frac{\frac{1}{a^x}-1}{(-x)^n\left(\frac{1}{a}+1\right)}$$

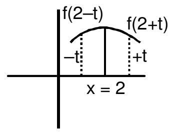
$$= \frac{-(a^x-1)}{(a^x+1)(-x)^n}$$

$$\Rightarrow \quad f(-x) = f(x) \ \Rightarrow\ -(-x)^n = 1 \Rightarrow\ n = -\frac{1}{3}$$

Sol.49 B

even $f(-x) = f(x)$
Even function are symmetric about y-axis that means symmetric about $x = 0$ line.





$f(2-x) = f(2+x)$

Sol.50 B

$f(-x) = -f(x)$
By Definition
$f(x)$ is an odd function

Sol.51 C

$$g : [-2, 2] \to R\ ;\ g(x) = x^3 + \tan x + \left[\frac{x^2+1}{p}\right]$$

$$g(-x) = -g(x) \ \Rightarrow\ \left[\frac{x^2+1}{p}\right] = 0$$

$$\therefore \qquad 0 \le \frac{x^2+1}{p} < 1$$

$$\Rightarrow \qquad x^2 + 1 = 5 \ \Rightarrow\ \frac{5}{p} < 1 \ \Rightarrow\ p > 5$$

Sol.52 D

$$f(x) = \frac{xf(x^2)}{2+\tan^2 x.f(x^2)} \qquad \text{given that } f(-x) = f(x) \ ....(1)$$

$$f(-x) = \frac{-xf(x^2)}{2+\tan^2 x + f(x^2)} \Rightarrow f(-x) = -f(x) \quad ...(2)$$

When both conditions are there only one possibility is there when $f(x) = 0 \ \Rightarrow\ f(10) = 0$

Sol.53 C

$f(x) = \sec(\sin x)$
$f(T) = f(0)$ $\qquad n = 1$
$\sec(\sin T) = 1$ $\qquad T = \pi$
$\sin T = 0 \ \Rightarrow\ T = n\pi$

Sol.54 D

$$f(x) = \sin\sqrt{[a]}\,x \ \Rightarrow\ \frac{2\pi}{\sqrt{[a]}} = \pi \quad \Rightarrow\ 4 = [a]$$

$\therefore\ a \in [4, 5)$

Sol.55 A

$f(x) = x + a - [x + b] + \sin\pi x + \cos 2\pi x + \sin 3\pi x + \cos 4\pi x + ... + \sin(2n-1)\pi x + \cos 2n\pi x$
$f(x) = x + a - (x + b) + \{x + b\} + \sin\pi x + \cos 2\pi x + \sin 3\pi x + \cos 4\pi x + ... + \sin(2n-1)\pi x + \cos 2n\pi x$
$\sin\pi x \ \ \cos 2\pi x \ \ \sin 3\pi x$

$$\text{Period } \frac{2\pi}{\pi}, \frac{2\pi}{2\pi}, \frac{2\pi}{3\pi}, \frac{2\pi}{4\pi}, \frac{2\pi}{5x} \cdots \frac{2\pi}{(2n-1)\pi}, \frac{2\pi}{2n\pi}$$

$$\frac{2}{1}, \frac{2}{2}, \frac{2}{3}, \frac{2}{4}\ \frac{2}{5} .... \left(\frac{2}{2n-1}\right), \left(\frac{2}{2n}\right)$$

$$\text{Period} = \frac{2}{1} = 2$$

Sol.56 C

$$f(x) = \sin\frac{\pi}{4}[x] + \cos\frac{\pi x}{2} + \cos\frac{\pi}{3}[x]$$

If $0 \le x < 1$ then $y = 0$

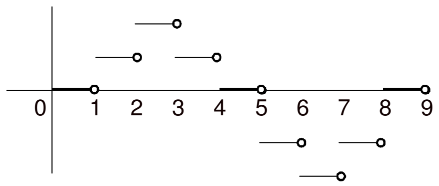
If $1 \le x < 2$ then $y = \frac{1}{\sqrt{2}}$

If $2 \le x < 3$ then $y = 1$

If $3 \le x < 4$ then $y = \frac{1}{\sqrt{2}}$

If $4 \le x < 5$ then $y = 0$

$\frac{2\pi}{\left(\frac{\pi}{4}\right)} = 8$



Period 8/4/6 $\Rightarrow L_{cm} = 24$

Sol.57 A

$f(x) = [x] + \left[x+\frac{1}{3}\right] + \left[x+\frac{2}{3}\right] - 3x + 15$

$f\left(x+\frac{1}{3}\right) = \left[x+\frac{1}{3}\right] + \left[x+\frac{2}{3}\right] + [x+1] - 3\left(x+\frac{1}{3}\right) + 15$

$f\left(x+\frac{1}{3}\right) = \left[x+\frac{1}{3}\right] + \left[x+\frac{2}{3}\right] + [x] + 1 - 3x - 1 + 15$

$f\left(x+\frac{1}{3}\right) = [x] + \left[x+\frac{1}{3}\right] + \left[x+\frac{2}{3}\right] - 3x + 15$

$f\left(x+\frac{1}{3}\right) = f(x) \Rightarrow$ period $= \frac{1}{3}$

Sol.58 A

(A) $f(x) = \frac{16^x - 1}{4^x} = 4^x - \frac{1}{4^x}$

$f(-x) = 4^{-x} - \frac{1}{4^{-x}} = \frac{1}{4^x} - 4^x = -\left(4^x - \frac{1}{4^x}\right) = -f(x)$

$f(-x) = -f(x)$ odd

(B) $f(x) = \sin|x|$

$f(-x) = \sin|-x| = \sin|x| = f(x)$

Sol.59 C

$f(x) = x(2-x)$; $f(x+2) = f(x) \Rightarrow$ period $= 2$

Sol.60 C

$f(2, 4) \to (1, 3)$

$2 < x < 4$; $1 < \frac{x}{2} < 2 \Rightarrow \left[\frac{x}{2}\right] = 1$

$y = f(x) = x - 1 \Rightarrow x = y + 1$

$f^{-1}(y) = y + 1$; $f^{-1}(x) = x + 1$

Sol.61 B

$f : R \to R$; $f(x) = x^3 + ax^2 + bx + c$

$f'(x) = 3x^2 + 2ax + b > 0$

$D \le 0 \Rightarrow 4a^2 - 4(3)b \le 0$

$\Rightarrow a^2 - 3b \le 0 \Rightarrow a^2 \le 3b$

EXERCISE – II HINTS & SOLUTIONS

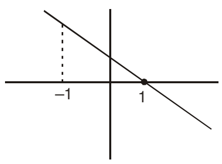
Sol.1 A,B

$f : [-1, 1] \to [0, 2]$

$y = 1 - x$ one-one - into

$y = 0$ $\quad y = 1 + x$

$y = 2$ $\quad \begin{matrix} x = 1 & y = 2 \\ x = -1 & y = 0 \end{matrix}$



Range = Co-domain

Sol.2 B,C,D

(A) $y = \sin(\sin^{-1} x) = x$ (Bijective function)

(B) $y = \frac{2}{\pi}[\sin^{-1}(\sin x)] = \frac{2x}{\pi}$

Range $\notin$ co-domain

Not a Bijective function

(C) $y = x \qquad 0 < x \le 1$

$= -x \quad -1 \le x < 0$

$= 0 \qquad x = 0$

Not a Bijective function

(D) $y = x^3 \qquad 0 < x \le 1$

$= -x^3 \quad -1 \le x < 0$

$= 0 \qquad x = 0$

Not a Bijective function

Sol.3 A,C

$f : N \to I$

$f(x) = \frac{n-1}{2}$ n is odd $n = 1, 3, 5, 7........$ (mapped to 0 1 2 3)

$= -\frac{n}{2}$ n is even $n = 2, 4, 6, 8........$ (mapped to –1 –2 –3 –4)

one-one onto.

Sol.4 A,C

$f(x) = \frac{1-x}{1+x}$; $g(x) = 4x(1-x)$

$$f[g(x)] = f(4x(1-x)) = \frac{1-4x(1-x)}{1+4x(1-x)} = \frac{1-4x+4x^2}{1+4x-4x^2}$$

$$g[f(x)] = g\left(\frac{1-x}{1+x}\right) = 4\left(\frac{1-x}{1+x}\right)\left(1-\frac{1-x}{1+x}\right) = \frac{8x(1-x)}{(1+x)^2}$$

Sol.5 A,B,C

By definition

Sol.6 A,B,C

$f(x) = \sin^4 3x + \cos^4 3x$

$f(T) = f(0)$

$\Rightarrow \sin^4 3T + \cos^4 3T = 1$

$\Rightarrow \sin^4 3T = 1 - \cos^4 3T = \sin^2 3T(1 + \cos^2 3T)$

$\Rightarrow \sin^2 3T(\sin^2 3T - 1 - \cos^2 3T) = 0$

$\therefore \sin^2 3T = 0$ or $\cos^2 3T - \sin^2 3T = -1$

$3T = n\pi$ or $\cos 6T = -1 \Rightarrow 6T = 2n\pi \pm \pi$

$T = \frac{n\pi}{3}$ or $T = (2n+1)\frac{\pi}{6}$

$n = 1$ $T = \frac{\pi}{3}$ or $n = 0$ $T = \frac{\pi}{6}$; $n = 1$ $T = \frac{3\pi}{6} = \frac{\pi}{2}$

Sol.7 B,C,D

$f : R \to [-1, 1]$

$0 \le x < 1$; $y = 0$

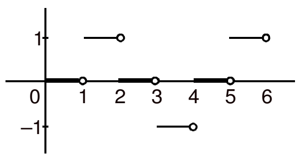
$1 \le x < 2$; $y = 1$

$2 \le x < 3$; $y = 0$

$3 \le x < 4$; $y = -1$

$4 \le x < 5$; $y = 0$

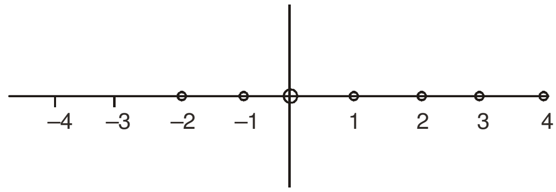
$5 \le x < 6$; $y = 1$



Sol.8 B,C,D

$f(x) = \frac{\sin \pi[x]}{\{x\}} = 0$ Range = 0

$x \notin I \Rightarrow x \in R - (I)$ $f(x) = 0\ \forall\ x \in R - I$



$$g(x) = \operatorname{sgn}\left(\operatorname{sgn}\left(\frac{\{x\}}{\sqrt{\{x\}}}\right)\right) - 1 = 1 - 1 = 0 \ ; x \in R - I$$

$g(x) = 0\ \forall\ x \in R - I$

Sol.9 A,D

$f(x) = \sin x + \tan x + \operatorname{sgn}(x^2 - 6x + 10) = 1$

$\Downarrow$ $\Downarrow$ $\Downarrow$

2π π $D = 36 - 40 < 0$

2π 2π

3π 3π

4π 4π

Sol.10 A,B,C,D

(A) $f(x) = \operatorname{sgn} e^{-x}$

(B) $f(x) = 1$; x is Rational

$= 0$; x is Irrational

(C) $f(x) = \sqrt{\frac{8}{1+\cos} + \frac{8}{1-\cos x}} = \frac{4}{|\sin x|}$

period $= \pi$

(D) $f(x) = \left[x + \frac{1}{2}\right] + \left[x - \frac{1}{2}\right] + 2[-x]$

$$= \left(x+\frac{1}{2}\right) - \left\{x+\frac{1}{2}\right\} + \left(x-\frac{1}{2}\right) + \left\{x-\frac{1}{2}\right\} + 2(-x) - 2\{-x\}$$

$$= 2x - 2x - \left\{x+\frac{1}{2}\right\} + \left\{x-\frac{1}{2}\right\} - 2\{-x\}$$

$$= -\left\{x+\frac{1}{2}\right\} + \left\{x-\frac{1}{2}\right\} - 2\{-x\}$$

EXERCISE – III HINTS & SOLUTIONS

Sol.1 (i) $f(x) = \dfrac{x^3 - 5x + 3}{x^2 - 1}$

$x^2 - 1 \neq 0 \Rightarrow x \neq \pm 1 \therefore x \in R - \{-1, 1\}$

(ii) $f(x) = \dfrac{1}{\sqrt{x + |x|}}$

$x + |x| > 0 \Rightarrow x > -|x| \therefore x \in (0, \infty)$

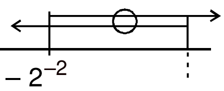
(iii) $f(x) = e^{x + \sin x} \quad \therefore x \in R$

(iv) $f(x) = \dfrac{1}{\log_{10}(1-x)} + \sqrt{x+2}$

$\log_{10}(1-x) \neq 0 \Rightarrow 1 - x \neq 1 \quad \Rightarrow x \neq 0$

$\&\ 1 - x \neq 0 \Rightarrow x \neq -1$

$\&\ 1 - x > 0 \Rightarrow x < 1$



$\&\ x + 2 \geq 0 \Rightarrow x \geq -2$

$\therefore x \in [-2, 0) \cup (0, 1)$

(v) $\log_x \log_2 \left(\dfrac{1}{x - \frac{1}{2}}\right) \Rightarrow \log_2 \left(\dfrac{1}{x - \frac{1}{2}}\right) > 0 ; x \neq 1$

$\Rightarrow \dfrac{1}{x - \frac{1}{2}} > 1 \ ; \ x < 0 \Rightarrow \dfrac{1}{x - \frac{1}{2}} - 1 > 0$

$\Rightarrow \dfrac{1 - x + \frac{1}{2}}{x + \frac{1}{2}} > 0 \Rightarrow \dfrac{x - \frac{3}{2}}{x - \frac{1}{2}} < 0$

$\therefore x \in \left(\frac{1}{2}, 1\right) \cup \left(\frac{3}{2}, 1\right)$

(vi) $f(x) = \sqrt{3 - 2^x - 2^{1-x}}$

$\Rightarrow 3 - 2^x - 2^{1-x} \geq 0 \Rightarrow 3 \geq 2^x + \dfrac{2}{2^x}$ (Let $t = 2^x$)

$\Rightarrow 3 \geq \dfrac{t^2 + 2}{2^t} \quad \Rightarrow t^2 + 2 \leq 3t$

$\Rightarrow t^2 + 2 < 3t \quad \Rightarrow t^2 - 3t + 2 \leq 0$

$\Rightarrow 1 \leq t \leq 2 \quad \Rightarrow 1 \leq 2^x \leq 2$

$\therefore 0 \leq x \leq 1$

(vii) $f(x) = \sqrt{1 - \sqrt{1 - x^2}}$

$1 - \sqrt{1 - x^2} \geq 0 \Rightarrow 1 \geq \sqrt{1 - x^2}$

$\Rightarrow 1 - x^2 \leq 1 \quad \Rightarrow -x^2 \leq 0 \Rightarrow x^2 \geq 0 \therefore x \in R$

$\&\ 1 - x^2 \geq 0 \Rightarrow x^2 - 1 \leq 0 \Rightarrow -1 \leq x \leq 1$

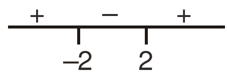
$\therefore x \in [-1, 1]$

(viii) $f(x) = (x^2 + x + 1)^{-3/2} = \dfrac{1}{(x^2 + x + 1)^{3/2}}$

$\therefore x \in R$

(ix) $f(x) = \sqrt{\dfrac{x-2}{x+2}} + \sqrt{\dfrac{1-x}{1+x}}$

$\dfrac{x-2}{x+2} \geq 0$



$\therefore x \in (-\infty, -2) \cup [2, \infty)$

$\&\ \dfrac{1-x}{1+x} \geq 0 \Rightarrow \dfrac{x-1}{x+1} \leq 0 \Rightarrow -1 < x \leq 1$

$\therefore x \in \phi$

(x) $f(x) = \sqrt{\tan x - \tan^2 x}$

$\Rightarrow \tan x - \tan^2 x \geq 0 \quad \Rightarrow \tan^2 x - \tan x \leq 0$

$\Rightarrow \tan x (\tan x - 1) \leq 0$

$0 \leq \tan x \leq 1 \qquad ; x \in \left[0, \frac{\pi}{4}\right]$

$0 \leq x \leq \frac{\pi}{4} \qquad ; x \in \left[n\pi, n\pi + \frac{\pi}{4}\right]$

(xi) $f(x) = \dfrac{1}{\sqrt{1 - \cos x}}$

$1 - \cos x > 0 \quad \cos x \neq 1$

$\cos x < 1 \quad x \neq 2n\pi$

$\therefore x \in R - \{2n\pi\}$

(xii) $f(x) = \sqrt{\log_{1/4}\left(\dfrac{5x - x^2}{4}\right)}$

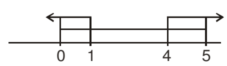
$\log_{1/4}\left(\dfrac{5x - x^2}{4}\right) \geq 0$

$\Rightarrow \dfrac{5x - x^2}{4} \leq 1 \Rightarrow \dfrac{x^2 - 5x + 4}{4} \geq 0$

$\Rightarrow (x-4)(x-1) \geq 0 \quad \therefore x \geq 4 \ ; \ x \leq 1$

$\&\ \dfrac{5x - x^2}{4} > 0 \Rightarrow x(x-5) < 0 \Rightarrow 0 < x < 5$

$\therefore x \in (0, 1] \cup [4, 5)$



(xiii) $f(x) = \log_{10}(1 - \log_{10}(x^2 - 5x + 16))$

$1 - \log_{10}(x^2 - 5x + 16) > 0$

$\Rightarrow \log_{10}(x^2 - 5x + 16) < 1 \Rightarrow x^2 - 5x + 16 < 10$

$\Rightarrow x^2 - 5x + 6 < 0 \qquad \therefore\ 2 < x < 3$

$\&\ x^2 - 5x + 6 > 0 \qquad \Rightarrow x \in R$

Domain $x \in (2, 3)$.

Sol.2 **(i)** $f(x) = |x - 3|$ is always positive $\therefore\ y \in [0, \infty)$

(ii) $f(x) = \dfrac{x}{1+x^2} = y$

$\Rightarrow x = y + yx^2 \quad \Rightarrow yx^2 - x + yx^2$

$D \geq 0 \quad \Rightarrow 4y^2 - 1 \leq 0 \ \Rightarrow -\dfrac{1}{2} \leq y \leq \dfrac{1}{2}$

Aliter (1) : $y = \dfrac{x}{1+x^2}$

$D_f\ x \in R$

$x = 0$, $y = 0$

$x \neq 0$, $y = \dfrac{1}{x + \frac{1}{x}}$

$x + \dfrac{1}{x} \geq 2$

$x + \dfrac{1}{x} \leq -2$

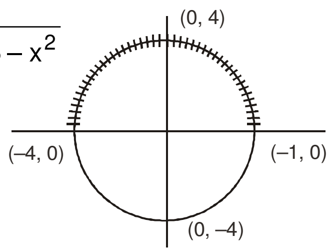
$\Rightarrow \left|x + \dfrac{1}{x}\right| \geq 2 \ \Rightarrow \dfrac{1}{\left|x + \frac{1}{x}\right|} \leq \dfrac{1}{2}$

$\Rightarrow -\dfrac{1}{2} \leq \dfrac{1}{\left|x + \frac{1}{x}\right|} \leq \dfrac{1}{2} \ \Rightarrow y \in \left[-\dfrac{1}{2}, \dfrac{1}{2}\right]$

Aliter (2) : $y = \dfrac{x}{1+x^2}$; $x \in R$

$x = \tan\theta \qquad \Rightarrow y = \dfrac{1}{2}\sin 2\theta = \left[-\dfrac{1}{2}, \dfrac{1}{2}\right]$.

(iii) $f(x) = \sqrt{16 - x^2}$



$y = \sqrt{16 - x^2}$

$y \in [0, 4]$

Half part of circle

$x^2 = 16 - y^2 \geq 0$

$\Rightarrow y^2 - 16 \leq 0$

$-4 \leq y \leq 4$ But y can take only +ve value

$0 \leq y \leq 4$

(iv) $f(x) = \dfrac{|x - 4|}{x - 4}$

$x \geq 4\ \ y = 1$; $x < 4\ \ y = -1 \Rightarrow y \in \{-1, 1\}$

(v) $f(x) = 5 + 3\sin x + 4\cos x$

$-5 \leq 3\sin x + 4\cos x \leq 5$

$\Rightarrow 0 \leq 5 + 3\sin x + 4\cos x \leq 10 \Rightarrow 0 \leq y \leq 10$

(vi) $f(x) = \dfrac{1}{1+\sqrt{x}} = y$

$\sqrt{x} = \dfrac{1-y}{y} \geq 0 \ \Rightarrow \dfrac{y-1}{y} \leq 0 \ \Rightarrow y \in (0, 1]$

(vii) $y = 2 - 3x + 5x^2 = -(5x^2 + 3x - 2)$

$-\left(\dfrac{-D}{4a}\right) = \dfrac{(9+40)}{20} = \dfrac{49}{20} \ \therefore\ y \in \left(-\infty, \dfrac{49}{20}\right]$

(viii) $y = 3\,|\sin x| - 4\,|\cos x|$

$y_{min} =$ min max

$y_{min} = 0 - 4 = -4$

$y_{max} = 3(1) - 4(0) = 3 \quad \therefore\ y \in [-4, 3]$

(ix) $y = \dfrac{\sin}{\sqrt{1+\tan^2 x}} + \dfrac{\cos x}{\sqrt{1+\cot^2 x}}$ $\quad \cos x \neq 0$, $\sin x \neq 0$

$= \dfrac{\sin x}{|\sec x|} + \dfrac{\cos x}{|\operatorname{cosec} x|}$

$= \sin x\,|\cos x| + |\cos x|\,\sin x|$

cos x & sin x can't be on vertical & Horizontal line.

Case – I : $0 < x < \dfrac{\pi}{2} \Rightarrow 0 < 2x < \pi$

$f(x) = \sin 2x \in (0, 1]$

Case – II : $x \in \left(\dfrac{\pi}{2}, \pi\right)$

$f(x) = -\sin x\cos x + \sin x\cos x = 0$

Case – III : $\pi < x < \dfrac{3\pi}{2} \Rightarrow 2\pi < 2x < 3\pi$

$f(x) = -\sin x\cos x - \sin x\cos x = -\sin 2x$

$0 < \sin 2x \leq 1 \ \Rightarrow -1 \leq -\sin 2x < 0$

$\therefore\ f(x) \in [-1, 0)$

Case – IV : $\dfrac{3\pi}{2} < x < 2\pi$

$f(x) = 0 \ \therefore\ y \in [-1, 1]$

(x) $f(x) = 1 - |x - 2| \Rightarrow y = 1 - |x - 2|$

$\Rightarrow |x - 2| = 1 - y \geq 0 \qquad \therefore\ y \leq 1$

(xi) $y = \dfrac{1}{\sqrt{x-5}} \Rightarrow \sqrt{x-5} = \dfrac{1}{y} > 0 \ \therefore\ y \in R^+$

(xii) $f(x) = \dfrac{1}{2 - \cos 3x} = y \ \Rightarrow \dfrac{1}{y} = 2 - \cos 3x$

$\Rightarrow \cos 3x = 2 - \dfrac{1}{y}$

$\Rightarrow -1 \le 2 - \dfrac{1}{y} \le 1 \qquad \Rightarrow -3 \le -\dfrac{1}{y} \le -1$

$\Rightarrow 1 \le \dfrac{1}{y} \le 3 \;\Rightarrow\; \dfrac{1}{3} \le y \le 1$

(xiii) $y = \dfrac{x+2}{x^2-8x-4} \Rightarrow yx^2 - 8xy - 4y - x - 2 = 0$

$\Rightarrow yx^2 - x(8y+1) - (4y+2) = 0$

$D \ge 0 \quad \Rightarrow (8y+1)^2 + 4y(4y+2) \ge 0$

$\Rightarrow 80y^2 + 24y + 1 \ge 0$

$\Rightarrow (4y+1)(20y+1) \ge 0$

$\therefore y \in \left(-\infty, -\dfrac{1}{4}\right] \cup \left[-\dfrac{1}{20}, \infty\right)$

(xiv) $y = \dfrac{x^2-2x+4}{x^2+2x+4}$

$\Rightarrow yx^2 + 2xy + 4y = x^2 - 2x + 4$

$\Rightarrow (y-1)x^2 + 2\pi(y+1) + 4(y-1) = 0$

at $y = 1 \Rightarrow 2x(2) = 0 \Rightarrow x = 0$

$D \ge 0 \quad \Rightarrow 4(y+1)^2 - 16(y-1)^2 \ge 0$

$\Rightarrow (y-3)(3y-1) \le 0 \quad \therefore \dfrac{1}{3} \le y \le 3$

(xv) $f(x) = 3\sin\sqrt{\dfrac{\pi^2}{16} - x^2}$

$0 \le \dfrac{\pi^2}{16} - x^2 \le \dfrac{\pi^2}{16}$

$\Rightarrow 0 \le \sqrt{\dfrac{\pi^2}{16} - x^2} \le \dfrac{\pi}{4} \quad \Rightarrow 0 \le \theta \le \dfrac{\pi}{4}$

$\Rightarrow 0 \le \sin\theta \le \dfrac{1}{\sqrt{2}} \;\Rightarrow\; 0 < 3\sin\theta < \dfrac{3}{\sqrt{2}}$

(xvi) $f(x) = x^4 - 2x^2 + 5 = (x^2-1)^2 + 4$

min at $x = 1$, min. value $= 4$

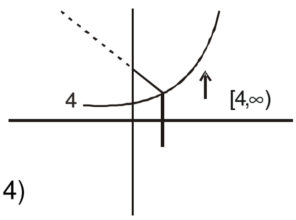
range $= [4, \infty)$

Aliter

$x^2 = -t \ge 0$

$t^2 - 2t + 5$

$\left(-\dfrac{b}{2a}, \dfrac{-D}{4a}\right) = (1, 4)$



(xvii) $f(x) = x^3 - 12x \qquad x \in [-3, 1]$

at $x = -3$; $y = -27 + 3 = 9$

at $x = 1$; $y = 1 - 12 = -11 \;\therefore\; y \in [-11, 9]$

(xviii) $f(x) = \sin^2 x + \cos^4 x = \sin^2 x + (1 - \sin^2 x)^2$

$= \sin^2 x + 1 + \sin^4 x - 2\sin^2 x = 1 + \sin^4 x - \sin^2 x$

$= \left(\sin^2 x - \dfrac{1}{2}\right)^2 + \dfrac{3}{4}$; $\sin^2 x = 0,\ y = 1$

min value $= \dfrac{3}{4}$; $\sin^2 x = 1,\ y = 1 \;\therefore\; \left[\dfrac{3}{4}, 1\right]$

Sol.3 **(i)** $f(x) = \dfrac{1}{\sqrt{4+3\sin x}} \qquad \Rightarrow y = \dfrac{1}{\sqrt{4+3\sin x}}$

$4 + 3\sin x > 0 \Rightarrow \sin x > -\dfrac{4}{3} \quad \therefore x \in R$

& $y^2 = \dfrac{1}{4+3\sin x} \quad \Rightarrow 3\sin x = \dfrac{1}{y^2} - 4$

$\Rightarrow 1 \le \dfrac{1}{y^2} \le 7 \Rightarrow \dfrac{1}{7} \le y^2 \le 1 \Rightarrow \dfrac{1}{\sqrt{7}} \le y \le 1$

(ii) $f(x) = x!$

Domain $= N \cup \{0\}$

Range $= n!,\ \{n = 0, 1, 2, 3, ...\}$

(iii) $f(x) = \dfrac{x^2-9}{x-3}$

$y = x + 3 \quad x \ne 3$

Domain $R - \{3\}$; Range $R - \{6\}$

(iv) $f(x) = \sin^2(x^3) + \cos^2(x^3)$

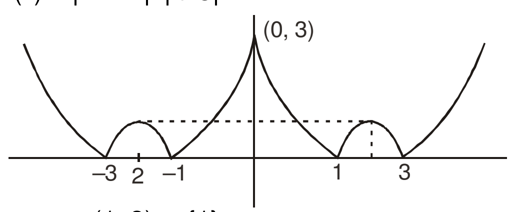
Domain $x \in R$; Range $= \{1\}$

Sol.4 $f(x) = \dfrac{4^x}{4^x+2}$

$f(1-x) = \dfrac{4^{1-x}}{4^{1-x}+2} = \dfrac{4}{4+2.4^x} = \dfrac{2}{2+4^x}$

$\Rightarrow f(x) + f(1-x) = \dfrac{4^x}{4^x+2} + \dfrac{2}{4+2^x} = 1$

Sol.5 $f(x) = |x^2 - 4|x| + 3|$



$\Rightarrow a \in (1, 3) \cup \{1\}$

Sol.6 $2x + 3[x] - 4\{-x\} = 4$

Case–I : $x \in I$

$2x + 3x - 0 = 4 \Rightarrow x = \dfrac{4}{5}$ (reject)

Case–II : $x \notin I$

$2x + 3[x] - 4(1 - \{x\}) = 4$

$2x + 3[x] - 4 + 4\{x\} = 4$

$2x + 3[x] - 4 + 4(x - [x]) = 4$

$2x + 3[x] - 4 + 4x - 4[x] = 4$

$\boxed{6x = 8 + [x]}$ ...(1)

$2x + 3(x - \{x\}) - 4 + 4\{x\} = 4$

$2x + 3x - 3\{x\} - 4 + 4\{x\} = 4$

$5x + \{x\} = 8$

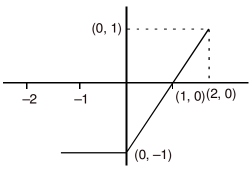
$0 \le \{x\} < 1 \Rightarrow 0 \le 8 - 5x < 1$

$\Rightarrow -8 \le -5x < -7 \quad \Rightarrow \dfrac{7}{5} < x \le \dfrac{8}{5}$

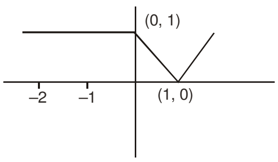
$[x] = 1 \Rightarrow 6x = 8 + 1 \Rightarrow x = \dfrac{9}{6} = \dfrac{3}{2}.$

Sol.7 $f(x) \to [-2, 2]$

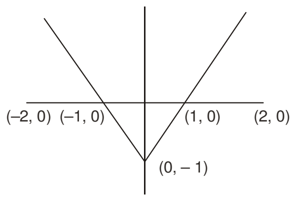
$$f(x) = \begin{cases} -1 \ ; & -2 \le x \le 0 \\ x - 1 \ ; & 0 \le x \le 2 \end{cases}$$



$$|f(x)| = \begin{cases} 1 \ ; & -2 \le x \le 0 \\ -x + 1 \ ; & 0 \le x \le 1 \\ x - 1 \ ; & -1 \le x \le 2 \end{cases}$$



$$f(|x|) = \begin{cases} -x - 1 \ ; & -2 \le x \le 0 \\ x - 1 \ ; & 0 \le x \le 1 \\ x - 1 \ ; & 1 \le x \le 2 \end{cases}$$



$$\therefore \ f(|x|) + |f(x)| = \begin{cases} -x \ ; & -2 \le x \le 0 \\ 0 \ ; & 0 \le x \le 1 \\ 2(x - 2) \ ; & 1 \le x < 2 \end{cases}$$

Sol.8 **(i)** $f(x) = \sqrt{x^2}$ & $g(x) = (\sqrt{x})^2$

Domain = $x \in R$; $x \in R^+ \cup \{0\}$

(ii) $f(x) = \sec(\sec^{-1} x)$ $g(x) = \text{cosec}(\text{cosec}^{-1} x)$

Domain are same

(iii) $f(x) = \sqrt{\dfrac{1 + \cos 2x}{2}}$; $g(x) = \cos x$

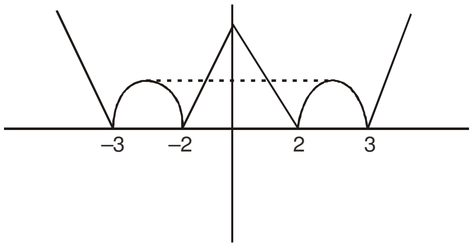
$f(x) = |\cos x|$; $g(x) = \cos x$

Appearance is not same.

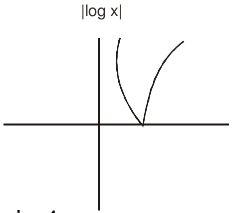
(iv) $f(x) = x$ & $g(x) = e^{\ell nx}$

Domain = $x \in R$ Domain = $x \in R^+$

Sol.9 **(i)** $f(x) = |x^2 + 5x + 6| = |(x+3)(x+2)|$ = Many one



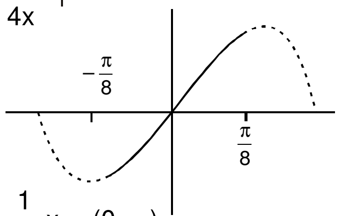
(ii) $|\log x| \Rightarrow$



Many-one.

(iii) $f(x) = \sin 4x$

$x \in \left(\dfrac{-\pi}{8}, \dfrac{\pi}{8}\right)$



one-one

(iv) $f(x) = x + \dfrac{1}{x}$ $x \in (0, \infty)$

$f(x) \ge 2$

$f'(x) = 1 - \dfrac{1}{x^2} = 0$

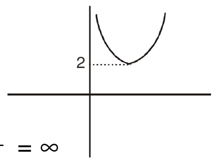
$x^2 = 1 \Rightarrow x = \pm 1$

$x = 1 \Rightarrow f'(x) = 0$

Curve turned because of $f'(x) = 0$

So many-one.

$x \to 0^+$ $f(x) \to 0^+ + \dfrac{1}{0^+} \to \infty$



$x \to \infty$ $f(x) \to \infty + \dfrac{1}{\infty} = \infty$

Aliter :

$x = 2$; $f(2) = 2 + \dfrac{1}{2} = \dfrac{5}{2}$

$x = \dfrac{1}{2}$; $f\left(\dfrac{1}{2}\right) = \dfrac{1}{2} + 2 = \dfrac{5}{2}$

$f(2)$ & $f\left(\dfrac{1}{2}\right)$ are same so many one.

(v) $f(x) = \sqrt{1 - e^{\left(\frac{1}{x} - 1\right)}}$

$1 - e^{\left(\frac{1}{x} - 1\right)} \ge 0 \Rightarrow \dfrac{1}{x} - 1 \le 0$

$\Rightarrow \quad \dfrac{1-x}{x} \le 0 \qquad \Rightarrow \quad \dfrac{x-1}{x} \ge 0$

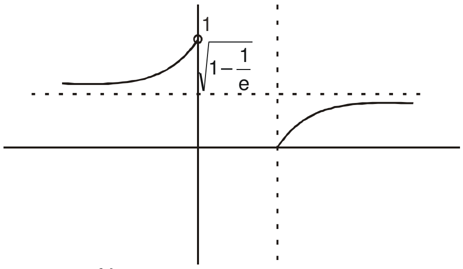
$\therefore\ x \in (-\infty, 0) \cup [1, \infty)$

$f(1) = \sqrt{1-e^{1-1}} = 0$

$\underset{x \to 0^-}{\text{Lt}} \sqrt{1-e^{\frac{1}{x}-1}} = 1$

$\underset{x \to \infty}{\text{Lt}} \sqrt{1-e^{\frac{1}{x}-1}} = \sqrt{1-\dfrac{1}{e}}$

$\underset{x \to -\infty}{\text{Lt}} \sqrt{1-e^{\frac{1}{x}-1}} = \sqrt{1-\dfrac{1}{e}}$



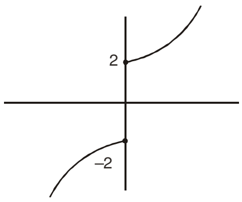
f is one-one.

(vi) $f(x) = \dfrac{3x^2}{4\pi} - \cos \pi x$

$f(-x) = \dfrac{3x^2}{4\pi} - \cos \pi x = f(x)$

even function so many-one

Sol.10 (i) $f(x) = \dfrac{1+x^6}{x^3}$



Domain $x \in R - \{0\}$

$f(x) = x^3 + \dfrac{1}{x^3}$

If $x \ge 0$ $f(x) \ge 2$; $x < 0$ $f(x) < -2$

Range $\ne$ co-domain $\Rightarrow$ Into function.

(ii) $f(x) = x \cos x$

$\quad\ \downarrow \qquad \downarrow$

$x \in R$; $[-1, 1]$

Range = co-domain $\Rightarrow$ onto

(iii) $f(x) = \dfrac{1}{\sin\sqrt{|x|}}$

$x \in R - \{0\}$; $y \in R - \{0\}$

Range $\ne$ Co-domain $\Rightarrow$ Into

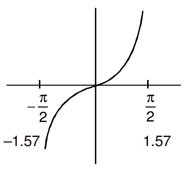
(iv) $\tan(2 \sin x)$

$-1 \le \sin x \le 1$

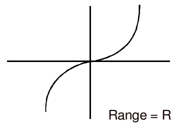
$-2 \le 2 \sin x \le 2$

$-\tan 2 \le (2 \sin x) \le \tan 2$

$-0.035 \qquad\qquad 0.035$



Sol.11 (i) $f(x) = x|x|$



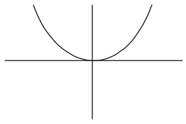
$x \ge 0$ $f(x) = x^2$

$x < 0$ $f(x) = -x^2$

Range = co-domain

one-one, onto

(ii) $f(x) = x^2$



many one, into

(iii) $y = \dfrac{x^2}{1+x^2} \ge 0$ always

$f(x) = f(-x) \Rightarrow$ many-one, into

(iv) $f(x) = x^3 - 6x^2 + 11x - 6$

$f'(x) = 3x^2 - 12x + 11$

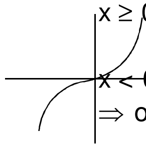
$D \ge 0$, Range = co-domain $\Rightarrow$ onto

Sol.12 (i) $x - \sin x$

$f'(x) = 1 - \cos x \ge 0 \Rightarrow$ many one.

(ii) $y = x|x|$

$x \ge 0$ $y = x^2$

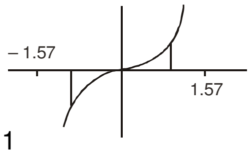


$x < 0$ $y = -x^2$

$\Rightarrow$ one-one & onto

(iii) $\tan \dfrac{\pi x}{4}$

$x = 1$; $y = \tan \dfrac{\pi}{4} = 1$



$x = -1$; $y = -1$

$\Rightarrow$ one-one-onto

(iv) $y = x^4 \Rightarrow$ many-one

Sol.13 (i) $f(x) = e^x$; $g(x) = \log x$

$fog(x) = f[g(x)] = f[\log x] = e^{\log x}$

$gof(x) = g[f(x)] = g(e^x) = \log e^x = x \log e$

(ii) $fog(x) = f[g(x)] = f[\sin x] = |\sin x|$

$gof(x) = g[f(x)] = g(|x|) = \sin |x|$

(iii) $fog(x) = f[g(x)] = f[x^2] = \sin^{-1} x^2$

$gof(x) = g[f(x)] = g(\sin^{-1} x) = (\sin^{-1} x^2)^2$

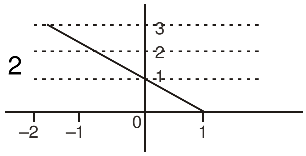
(iv) $fog(x) = f[g(x)] = f\left(1 - \dfrac{1}{1-x}\right) = \dfrac{3x^2 - 4x + 2}{(1-x)^2}$

$gof(x) = g[f(x)] = g(x^2 + 2) = \dfrac{x^2+2}{x^2+1}$

Sol.14 $f(x) = 1 + x^2$; $x \le 1$

$= x + 1$; $1 < x \le 2$

$g(x) = 1 - x$; $-2 \le x \le 1$



$f[g(x)] = 1 + g(x)]^2$; $g(x) \le 1$

$= g(x) + 1$; $1 < g(x) \le 2$

$fog(x) = f[g(x)] = 1 + [g(x)]^2$; $g(x) \le 1$

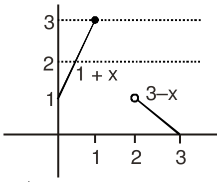
$= g(x) + 1\ ;\ 1 < g(x) \le 2$

$f[g(x)] = 1 + [g(x)]^2 = x^2 - 2x + 2\ ;\ 0 \le x \le 1$

$= g(x) + 1 = 2 - x\ ;\ -1 \le x < 0$

Sol.15 $f(x) = 1 + x\ ;\ 0 \le x \le 2$

$= 3 - x\ ;\ 2 < x \le 3$



$f[f(x)] = 1 + f(x)\ ;\ 0 \le f(x) \le 2$

$= 3 - f(x)\ ;\ 2 < f(x) \le 3$

$= 1 + 1 + x = 2 + x\ ;\ 0 \le x \le 1$

$= 4 - x\ ;\ 2 < x \le 3$

$= 3 - (1 + x) = 2 - x\ ;\ 1 < x \le 2$

Sol.16 $f(x) = \ell n\,(x^2 - x + 2) : R^+ \to R$

$g(x) = \{x\} + 1 : [1, 2] \to [1, 2]$

$f(g(x)) = f(\{x\} + 1)\quad a = \{x\} = \ell n\,[(a + 1)^2 - (a + 1) + 2]$

$= \ell n\,(a^2 + 2a + 1 - a - 1 + 2) = \ell n\,(a^2 + a + 2)$

domain of $f(g(x))$ = domain of $g(x) = [1, 2]$

$0 \le \{x\} < 1 \quad \Rightarrow\ 0 \le \{x\}^2 < 1$

$\Rightarrow\ 0 \le \{x\}^2 + \{x\} < 2 \quad \Rightarrow\ 2 \le a^2 + a + 2 < 4$

$\Rightarrow\ \ell n\,2 \le \ell n(a^2 + a + 2) < \ell n 4$

$\therefore$ Range $\in [\ell n 2, \ell n 4]$

Sol.17 $f(x).f\left(\dfrac{1}{x}\right) = f(x) + f\left(\dfrac{1}{x}\right)$

$f(x) = 1 \pm x^n$

$f(3) = 1 \pm (3)^n = -26$

$$\Rightarrow \begin{cases} 1 + 3^n = -26 \ \Rightarrow\ 3^n = -27 \\ 1 - 3^n = -26 \ \Rightarrow\ 3^n = 27 \ \Rightarrow\ n = 3 \end{cases}$$

$f(x) = 1 - x^3 \ \Rightarrow\ f'(x) = -3x^2 \ \Rightarrow\ f'(1) = -3$

Sol.18 $f(x + y) = f(x).f(y)\ ;\ f(1) = 2$

$f(x) = a^x \ \Rightarrow\ f(1) = a = 2 \ \Rightarrow\ f(x) = 2^x$

$$\sum_{n=1}^{10} f(n) = 2 + 2^2 + 2^3 + \ldots\ldots\ldots + 2^{10}$$

$$= 2\left(\frac{2^{10} - 1}{2 - 1}\right) = (2^{10} - 1)2 = 2\,(1023) = 2046$$

Sol.19 $f(x) = x^2 + \sin x\ ;\ 0 \le x < 1$

$= x + e^{-x}\ ;\ x \ge 1$

(i) An even function : $f(-x) = f(x)$

$f(x) = x^2 - \sin x\ ;\ -1 < x \le 0$

$= -x + e^x\ ;\ x \le -1$

(ii) odd function : $f(x) = -f(-x)$

$f(x) = -x^2 + \sin x\ ;\ -1 < x \le 0$

$= x - e^x\ ;\ x \le -1$

Sol.20 **(i)** $f(x) = \dfrac{(1 + 2^x)^7}{2^x}$

$f(-x) = \dfrac{(1 + 2^{-x})^7}{2^{-x}}$ none

(ii) $f(x) = \dfrac{\sec x + x^2 - 9}{x \sin x}$

$f(-x) = \dfrac{\sec x + x^2 - 9}{x \sin x}$ even

(iii) $f(x) = \sqrt{1 + x + x^2} - \sqrt{1 - x + x^2}$

$f(-x) = \sqrt{1 - x + x^2} - \sqrt{1 + x + x^2}$

$f(-x) = -f(x)$ odd

(iv) $f(x) = x|x|\ ;\ x \le -1$

$= [1 + x] - [x - 1] = 2\ ;\ -1 < x < 1$

$= -x\,|x|\ ;\ x \ge 1$

$f(-x) = -x|x|\ x \ge 1$

$= 2\ ;\ -1 < x < 1$

$= x\,|x|\ ;\ x \le -1.$ even

(v) $f(x) + f(-x) = \dfrac{2x\,(\sin x + \tan x)}{2\left[2 + \dfrac{x}{\pi}\right] - 3} + \dfrac{2x\,(\sin x + \tan x)}{2\left[2 - \dfrac{x}{\pi}\right] - 3}$

$$= 2x(\sin x + \tan x)\left\{\frac{1}{2\left[\dfrac{x}{\pi}\right] + 1} + \frac{1}{2\left[-\dfrac{x}{\pi}\right] + 1}\right\}$$

$$= 2x(\sin x + \tan x)\left\{\frac{2\left(\left[\dfrac{x}{\pi}\right] + \left[\dfrac{-x}{\pi}\right]\right) + 2}{\left(2\left[\dfrac{x}{\pi}\right] + 1\right)\left(2\left[\dfrac{-x}{\pi}\right] + 1\right)}\right\}$$

$$\left[\frac{x}{\pi}\right] + \left[\frac{-x}{\pi}\right] = \begin{cases} 0 & ;\ \dfrac{x}{\pi} \in n \ \text{ or } \ x \in n\pi \\ -1 & ;\ \dfrac{x}{\pi} \notin n \ \text{ or } \ x \notin n\pi \end{cases}$$

Case – I : $x = n\pi$

$f(x) + f(-x) = 0 \ \Rightarrow\ f(-x) = -f(x)$

Case – II : $x \ne n\pi$

$f(x) + f(-x) = 0 \ \Rightarrow\ f(-x) = -f(x)$

$f(x)$ is an odd function

Sol.21 **(i)** $f(x) = 2 + 3\cos(x - 2)$

Period $= 2\pi$ or $f(T) = f(0)$

(ii) $f(x) = \sin 3x + \cos^2 x + |\tan x|$

$$= \sin 3x + \frac{1 + \cos 2x}{2} + |\tan x|$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$= \frac{2\pi}{3} \qquad \frac{2\pi}{2} \qquad \pi$$

$$\frac{\text{LCM}}{\text{HCF}} = \frac{2\pi}{1} = 2\pi$$

(iii) $f(x) = \sin\frac{\pi x}{4} + \sin\frac{\pi x}{3}$

$$\downarrow \qquad \downarrow$$

$$\frac{2\pi}{\pi/4} = 8 \qquad \frac{2\pi}{\pi/3} = 6$$

$$\frac{\text{LCM}}{\text{HCF}} = \frac{8\times 3}{1} = 24$$

(iv) $f(x) = \cos\frac{3}{5}x - \sin\frac{2}{7}x$

$$\downarrow \qquad \downarrow$$

$$\frac{2\pi}{3/5} = \frac{10\pi}{3} \qquad \frac{2\pi}{2/7} = 7\pi$$

$$\frac{\text{LCM}}{\text{HCF}} = \frac{70\pi}{1} = 70\pi$$

(v) $f(x) = [\sin 3x] + |\cos 6x|$

period of $[\sin 3x] = \frac{2\pi}{3}$; period of $|\cos x| = \frac{\pi}{6}$

$\therefore$ period of $f(x) = \frac{2\pi}{3}$

(vi) $f(x) = \frac{1}{1-\cos x} = \frac{1}{2\sin^2\frac{x}{2}}$

period of $f(x) = \frac{\pi}{(1/2)} = 2\pi$

(vii) $f(x) = \frac{\sin 12x}{1+\cos^2 6x} = \frac{\sin 12x}{\frac{1}{3}+\frac{1}{2}\cos 12x}$

Period of $\sin 12x = \frac{2\pi}{12} = \frac{\pi}{6}$

Period of $\cos 12x = \frac{2\pi}{12} = \frac{\pi}{6}$ $\therefore$ Period $= \frac{\pi}{6}$

(viii) $f(x) = \sec^2 x + \operatorname{cosec}^3 x = \frac{1}{\cos^2 x} + \frac{1}{\sin^3 x}$

$$\downarrow \qquad \downarrow$$

$$\pi \qquad 2\pi$$

Period $= 2\pi$

Sol.22 **(i)** $f(x) = 1 - \frac{\sin^2 x}{1+\cot x} - \frac{\cos^2 x}{1+\tan x}$

$= 1 - 1 + \sin x\cos x = \frac{1}{2}\sin 2x$

$\therefore \frac{2\pi}{2} = \pi$ period

(ii) $f(x) = \log(2 + \cos 3x)$

$f(T) = f(0)$

$\Rightarrow \log(2 + \cos 3T) = \log 3 \Rightarrow 2 + \cos T = 3$

$\Rightarrow \cos T = 1 \Rightarrow 3T = 2\pi \quad \Rightarrow T = 2\pi/3$

$\therefore f\left(x + \frac{2\pi}{3}\right) = f(x)$

(iii) $f(x) = \tan\frac{\pi}{2}[\pi]$

$f(T) = f(0) \Rightarrow \tan\frac{\pi}{2}[T] = 0$

$\Rightarrow \frac{\pi}{2}[T] = \pi \Rightarrow [T] = 2 \Rightarrow T = 2$

(iv) $f(x) = e^{\ell n \sin x} + \tan^3 x - \operatorname{cosec}(3x-5)$

$= \sin x + \tan^3 x - \operatorname{cosec}(3x-5)$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$2\pi \qquad \pi \qquad 2\pi/3$$

$$\frac{\text{LCM}}{\text{HCF}} = \frac{2\pi}{1} = 2\pi$$

(v) $f(x) = \frac{1}{2}\left(\frac{|\sin x|}{\cos x} + \frac{\sin x}{\frac{1}{2}}\right)$

period of $|\sin x| = \pi$

period of $\sin x = 2\pi$

period of $\cos x = 2\pi$

period $= 2\pi$

(vi) $f(x) = \sin x + \tan\frac{x}{2} + \sin\frac{x}{2^2} + \tan\frac{x}{2^3}$

$$\downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow$$

Period $\quad 2\pi \quad \pi \quad 2^3\pi \quad 2^3\pi$

$+ \ldots\ldots\ldots + \sin\frac{x}{2^{n-1}} + \tan\frac{x}{2^n}$

$$\downarrow \qquad \downarrow$$

$$2^n\pi \qquad 2^n\pi$$

LCM of $(2\pi, 2\pi, 2^3\pi, 2^3\pi \ldots\ldots\ldots 2^n\pi, 2^n\pi) = 2^n\pi$

(vii) $f(x) = \frac{\sin x + \sin 3x}{\cos x + \cos 3x} = \frac{2\sin 2x\cos x}{2\cos 2x\cos x}$

period $= \pi$

Sol.23 **(i)** $f(x+p) = 1 + \{1 - 3f(x) + 3f^2(x) - f^3(x)\}^{1/3}$

$(f(x+p) - 1)^3 = 1 - 3f(x) + 3f^2(x) - f^3(x)$

$(1 - f(x+p))^3 = (f(x) - 1)^3$

replace $x \to x + p$

$(1 - f(x+2p))^3 = (f(x+p) - 1)^3$

$(1 - f(x+2p))^3 = -(f(x) - 1)^3 = (1 - f(x))^3$

$f(x+2p) = f(x)$ period $= 2p$

(ii) $f(x-1) + f(x+3) = f(x+1) + f(x+5)$....(1)

Replace x by x + 2

$f(x+1) + f(x+5) = f(x+3) + f(x+7)$ ....(2)

Add (1) and (2) we get $f(x-1) = f(x+7)$

replace $x \to x + 1$

$f(x) = f(x+8)$ period = 8

Sol.24 $f : R \to R$; $f(x) = \dfrac{e^{2x} - e^{-2x}}{2}$

$f(x) = \dfrac{e^{2x} - e^{-2x}}{2} \le -1$ one-one onto

$f'(x) = \dfrac{2e^{2x}}{2} - \dfrac{(-2)}{2} e^{-2x} = e^{2x} + e^{-2x} > 0$ one-one onto.

$y = \dfrac{e^{2x} - e^{-2x}}{2} \Rightarrow 2y = t - \dfrac{1}{t}$

$\Rightarrow t^2 - 2y\,t - 1 = 0 \Rightarrow t = \dfrac{2y \pm \sqrt{4y^2 + 4y}}{2}$

$\therefore e^{2x} = y \pm \sqrt{y^2 + 1} \Rightarrow 2x = \ell n\,(y \pm \sqrt{y^2+1}\,)$

$\Rightarrow x = \dfrac{1}{2}\,\ell n\,(y + \sqrt{y^2+1}\,)$

$\therefore f^{-1}(x) = \dfrac{1}{2}\ell n(x + \sqrt{x^2+1})$

Sol.25 $f : \left[-\dfrac{\pi}{3}, \dfrac{\pi}{6}\right] \to B$

$f(x) = 2\cos^2 x + \sqrt{3}\ \sin 2x + 1$

$= 1 + \cos 2x + \sqrt{3}\ \sin 2x + 1 = 2 + 2\sin\left(2x + \dfrac{\pi}{6}\right)$

$\because -1 \le \sin\left(2x + \dfrac{\pi}{6}\right) \le 1$

$\Rightarrow -2 \le 2\sin\left(2\pi + \dfrac{\pi}{6}\right) \le 2 \Rightarrow 0 \le f(x) \le 4$; $B \in [0, 4]$

$\therefore 2\sin\left(2x + \dfrac{\pi}{6}\right) = y - 2$

$\Rightarrow \sin\left(2x + \dfrac{\pi}{6}\right) = \dfrac{1}{2}(y - 2)$

$\Rightarrow 2x + \dfrac{\pi}{6} = \sin^{-1}\left(\dfrac{y-2}{2}\right)$

$\Rightarrow f^{-1}(y) = x = \dfrac{1}{2}\sin^{-1}\left(\dfrac{y-2}{2}\right) - \dfrac{\pi}{12}$

Sol.26 $f : N \to N$; $f(x) = x + (-1)^{x-1}$

Case–I : $x = 2K$; $K \in N$

$f(x) = x - 1 \Rightarrow y = x - 1$

$f^{-1}(x) = x + 1 \Rightarrow f^{-1}(x) = x - (-1)^{x-1}$

Case–II : $x = 2K + 1$; $K \in N$

$f(x) = x + 1 \Rightarrow y = x + 1 \Rightarrow x = y - 1$

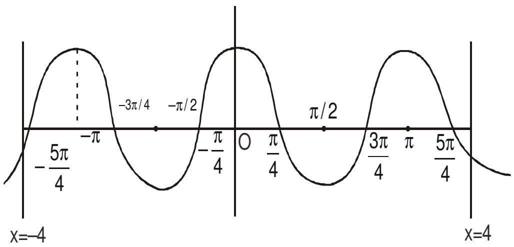
$\Rightarrow f^{-1}(x) = x - 1 \Rightarrow f^{-1}(x) = x - (-1)^{x-1}$

EXERCISE – IV HINTS & SOLUTIONS

Sol.1 (i) $f(x) = \sqrt{\cos 2x} + \sqrt{16 - x^2}$

$\Rightarrow 16 - x^2 \geq 0 \Rightarrow x^2 - 16 \leq 0 \Rightarrow -4 \leq x \leq 4.$

& $\cos 2x \geq 0$



$x \in \left[-\frac{5\pi}{4}, -\frac{3\pi}{4}\right] \cup \left[-\frac{\pi}{4}, \frac{\pi}{4}\right] \cup \left[\frac{3\pi}{4}, \frac{5\pi}{4}\right]$

(ii) $f(x) = \log_7 \log_5 \log_2 (2x^3 + 5x^2 - 14x)$

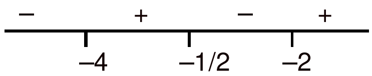
$\Rightarrow \log_5 \log_3 \log_2 (2x^3 + 5x^2 - 14x) > 0$

$\Rightarrow \log_3 \log_2 (2x^3 + 5x^2 - 14x) > 1$

$\Rightarrow \log_2 (2x^3 + 5x^2 - 14x) > 3$

$\Rightarrow 2x^3 + 5x^2 - 14x - 8 > 0$

$\Rightarrow (x - 2)(2x + 1)(x + 4) > 0$



$\therefore x \in \left(-4, -\frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$

(iii) $f(x) = \ell n\left(\sqrt{x^2 - 5x - 24} - x - 2\right)$

$\sqrt{x^2 - 5x - 24} > x + 2$

& $x^2 - 5x - 24 \geq 0 \Rightarrow (x - 8)(x + 3) \geq 0$

$\Rightarrow x \geq 8\ ;\ x \leq -3$

Case–1 : $x + 2 < 0 \Rightarrow x < -2$

$\sqrt{x^2 - 5x - 24} > x + 2$

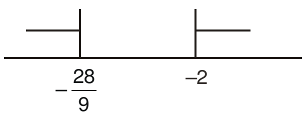
+ve +ve $\Rightarrow x < -2$

Case–2 : $x + 2 \geq 0 \Rightarrow x \geq -2$

$\sqrt{x^2 - 5x - 24} > (x + 2)$

$x^2 - 5x - 24 > x^2 + 4x + 4$

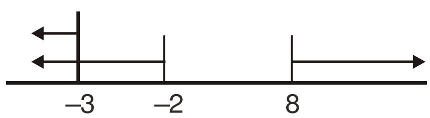
$x < -\frac{28}{9}$



Case–I ∪ Case–II $x \in \phi$

$x < -2$

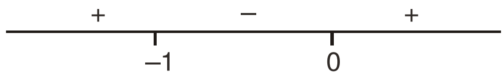
$x < -2 \cap (-\infty, -3] \cup [8, \infty) \quad \therefore x \in (-\infty, -3]$



(iv) $f(x) = \sqrt{\frac{1 - 5^x}{7^{-x} - 7}}$

$\frac{1 - 5^x}{7^{-x} - 7} \geq 0 \Rightarrow \frac{1 - 5^x}{1 - 7^{1+x}} \geq 0 \Rightarrow \frac{5^x - 1}{7^{1+x} - 1} \geq 0$

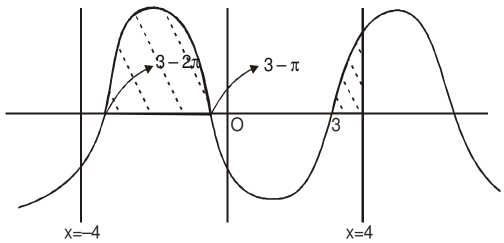
$5^x - 1 = 0 \Rightarrow x = 0$ & $7^{x+1} = 1 \Rightarrow x = -1$



(v) $y = \log_{10} \sin (x - 3) + \sqrt{16 - x^2}$

$16 - x^2 \geq 0 \Rightarrow x^2 - 16 \leq 0 \Rightarrow -4 \leq x \leq 4$

& $\sin (x - 3) > 0$



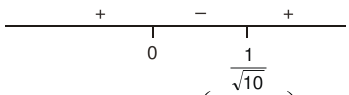
$\therefore x \in (3 - 2\pi, 3 - \pi) \cup (3, 4]$

(vi) $f(x) = \log_{100x}\left(\frac{2\log_{10} x + 1}{-x}\right)$

$100x > 0 \Rightarrow x > 0$

$100x \neq 1 \Rightarrow x \neq \frac{1}{100}$

$\frac{2\log_{10} x + 1}{-x} > 0$



$\Rightarrow \frac{2\log_{10} x + 1}{x} < 0 \quad \therefore x \in \left(0, \frac{1}{\sqrt{10}}\right)$

Aliter : $\frac{2\log_{10} x + 1}{-x} > 0\ ;\ x > 0$

$\Rightarrow 2\log_{10} x + 1 < 0 \Rightarrow x < \frac{1}{\sqrt{10}}$

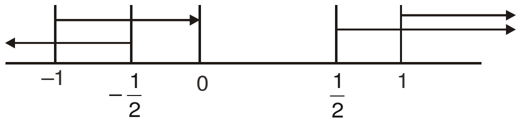
$\therefore x \in \left(0, \frac{1}{\sqrt{10}}\right) - \left\{\frac{1}{100}\right\}$

(vii) $f(x) = \frac{1}{\sqrt{4x^2 - 1}} + \ell n\, x(x^2 - 1)$

$4x^2 - 1 > 0 \Rightarrow x > \frac{1}{2}\ ;\ x < -1/2$

$x(x^2 - 1) > 0 \Rightarrow x(x+1)(x-1) > 0$

$x \in (-1, 0) \cup (1, \infty)$



$\therefore\ x \in \left(-1, -\frac{1}{2}\right) \cup (1, \infty)$

(viii) $f(x) = \sqrt{\log_{1/2}\left(\frac{x}{x^2-1}\right)}$

$\log_{1/2} \frac{x}{x^2-1} \geq 0 \Rightarrow \frac{x}{x^2-1} \leq 1 \Rightarrow \frac{x^2-x-1}{x^2-1} \geq 0$

$\therefore\ x \in (-\infty, -1) \cup \left[\frac{1-\sqrt{5}}{2}, 1\right) \cup \left[\frac{1+\sqrt{5}}{2}, \infty\right)$

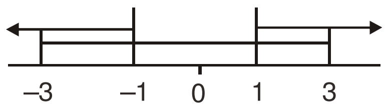
$\&\ \frac{x}{x^2-1} > 0 \Rightarrow \frac{x}{(x+1)(x-1)} > 0$

$\Rightarrow x \in (-1, 0) \cup (1, \infty)$

$\therefore\ x \in \left[\frac{1-\sqrt{5}}{2}, 0\right) \cup \left[\frac{1+\sqrt{5}}{2}, \infty\right)$

(ix) $f(x) = \sqrt{x^2 - |x|} + \frac{1}{\sqrt{9-x^2}}$

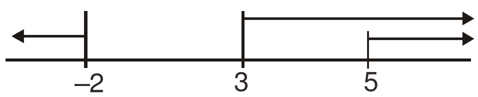
$x^2 - \lvert x\rvert \geq 0$	$9 - x^2 \geq 0$
$\lvert x\rvert \leq x^2$	$x^2 - 9 < 0$
$\lvert x\rvert \leq \lvert x\rvert^2$	$-3 < x < 3$
$\lvert x\rvert \leq 0$ or $\lvert x\rvert \geq 1$	
$x = 0$	$x \leq -1$ or $x \geq 1$



$\therefore\ x \in (-3, -1] \cup (1, 3) \cup \{0\}$

(x) $f(x) = \sqrt{(x^2 - 3x - 10)\ell n^2(x-3)}$

$x^2 - 3x - 10 \geq 0$	$x - 3 > 0$
$(x-5)(x+2) \geq 0$	$x > 3$
$x \geq 5\ x \leq -2$	$\ell n^2(x-3) = 0$
	$x - 3 = 1 \Rightarrow x = 4$



$\therefore\ x \in [5, \infty) \cup \{4\}$

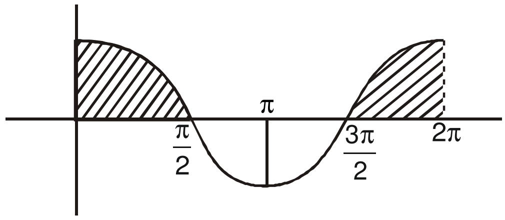
(xi) $f(x) = \sqrt{\log_x(\cos 2\pi x)}$

$\Rightarrow \log_x(\cos 2\pi x) \geq 0$

Case–1 : $0 < x < 1 \Rightarrow 0 < \cos 2\pi x \leq 1$

$\Rightarrow 0 < 2\pi x < 2\pi$ Let $t = 2\pi x$

$\Rightarrow 0 < t < 2\pi$



$\Rightarrow 0 \leq t < \frac{\pi}{2} \cup \frac{3\pi}{2} < t \leq 2\pi$

$\Rightarrow 0 \leq 2\pi x < \frac{\pi}{2} \cup \frac{3\pi}{2} < 2\pi x \leq 2\pi$

$\Rightarrow 0 \leq x < \frac{1}{4} \cup \frac{3}{4} < x \leq 1$

$\Rightarrow x \in \left[0, \frac{1}{4}\right) \cup \left(\frac{3}{4}, 1\right]\ \therefore\ x \in \left(0, \frac{1}{4}\right) \cup \left(\frac{3}{4}, 1\right)$

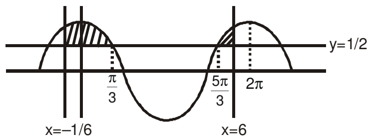
Case–2 : $x > 1 \Rightarrow \cos 2\pi x \geq 1$

$\Rightarrow \cos 2\pi x = 1 \Rightarrow 2\pi x = 2n\pi \Rightarrow x = n$

(xii) $f(x) = \frac{\sqrt{\cos x - \frac{1}{2}}}{\sqrt{6 + 35x - 6x^2}}$

$\Rightarrow 6 + 35x - 6x^2 > 0\ ;\ \cos x \geq \frac{1}{2}$

$\Rightarrow (x-6)(6x+1) < 0 \Rightarrow -\frac{1}{6} < x < 6$



$\therefore\ x \in \left(\frac{-1}{6}, \frac{\pi}{3}\right] \cup \left[\frac{5\pi}{3}, 6\right)$

(xiii) $f(x) = \sqrt{\log_{1/3}(\log_4([x]^2 - 5))}$

$\Rightarrow \log_{1/3}\log_4([x]^2 - 5) \geq 0$

$\Rightarrow 0 < \log_4([x]^2 - 5) \leq 1$

$\Rightarrow 1 < [x]^2 - 5 \leq 4 \Rightarrow 6 < [x]^2 \leq 9$

$[x] \rightarrow$ is always give integer value so it square also give integer value so,

$\Rightarrow [x]^2 = 9 \qquad \Rightarrow [x] = \pm 3$

$\Rightarrow [x] = 3 \qquad \Rightarrow 3 \leq x < 4$

$\&\ [x] = -3 \qquad \Rightarrow -3 < x \leq -2$

$\therefore\ x \in [-3, -2) \cup [3, 4)$

(xiv) $f(x) = \underset{D_1}{\frac{1}{[x]}} + \underset{D_2}{\log_{(2\{x\}-5)}(x^2-3x+10)} + \underset{D_2}{\frac{1}{\sqrt{1-|x|}}}$

$D_1 \to R - [0, 1)$

$D_2 : \log_{(2\{x\}-5)}(x^2 - 3x + 10)$

$\& \ 2\{x\} - 5 > 0 \Rightarrow \{x\} > \frac{5}{2}$ Not possible

$\therefore \ x \in \phi$

(xv) $f(x) = \log_x \sin x$

$x > 0, x \neq 1, \sin x > 0$

$\& \ 0 < x < \pi \Rightarrow 2n\pi < x < 2nx + \pi$

$\Rightarrow 2n\pi < x < (2n + 1)\pi \qquad n \in I$

(xvi) $f(x) = \log_2\left(-\log_{1/2}\left(1+\frac{1}{\sin\left(\frac{x^{\circ}}{100}\right)}\right)\right)$

$+ \sqrt{\log_{10}(\log_{10} x) - \log_{10}(4 - \log_{10} x) - \log 10^3}$

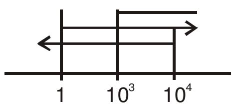
$\Rightarrow \log_{10}(\log_{10} x) - \log_{10}(4 - \log_{10} x) - \log_{10} 3 \geq 0$

$\log_{10}\left(\frac{\log_{10} x}{3(4-\log_{10} x)}\right) \geq 0$

$\Rightarrow \frac{\log_{10} x}{3(4-\log_{10} x)} \geq 1 \Rightarrow t \geq 12 - 3t$ (let $\log_{10} x = t$)

$\Rightarrow t \geq 3 \quad \Rightarrow \log_{10} x \geq 3 \quad \Rightarrow x \geq 10^3$

$\& \log_{10} x > 0 \Rightarrow x > 1$ and $4 - \log_{10} x > 0 \Rightarrow x < 10^4$



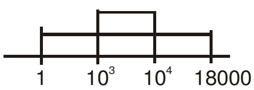
$\therefore \ x \in [10^3, 10^4)$

$-\log_{1/2}\left(1+\frac{1}{\sin\frac{x^{\circ}}{100}}\right) > 0 \Rightarrow \log_2\left(1+\frac{1}{\sin\frac{x^{\circ}}{100}}\right) > 0$

$\Rightarrow 1 + \frac{1}{\sin\frac{x^{\circ}}{100}} > 1 \quad \Rightarrow \frac{1}{\sin\frac{x\pi}{1800}} > 0$

$\Rightarrow \sin\frac{x\pi}{18000} > 0 \Rightarrow 0 < \frac{x\pi}{18000} < \pi$

$\Rightarrow 0 < x < 18000$



$\therefore \ x \in [10^3, 10^4)$

(xvii) $f(x) = \frac{1}{[x]} + \log_{1-\{x\}}(x^2 - 3x + 10)$

$+ \frac{1}{\sqrt{2-|x|}} + \frac{1}{\sqrt{\sec(\sin x)}}$

$D_1 : \ x \in R - [0, 1)$

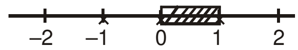
$D_2 : \ x^2 - 3x + 10 > 0 \ ; \quad 1 - \{x\} > 0 \ ; \quad 1 - \{x\} \neq 1$

$x \in R \qquad \{x\} < 1 \qquad [x] \neq 0$

$x \in R \qquad x \neq I$

$D_3 : \quad 2 - |x| > 0 \Rightarrow |x| < 2 \Rightarrow -2 < x < 2$

$D_4 : x \in R$



$\therefore \ x \in (-2, -1) \cup (-1, 0) \cup (1, 2)$

(xviii) $f(x) = \underset{D_1}{\sqrt{(5x-6-x^2)\,[\{\ell n\{x\}\}]}} + \underset{D_2}{\sqrt{(7x-5-2x^2)}} + \underset{D_3}{\left(\ell n\left(\frac{7}{2}-x\right)\right)^{-1}}$

$D_1 : \quad x \in R - I$

$\& \ 5x - 6 - x^2 \geq 0 \Rightarrow (x - 2)(x - 3) \leq 0 \Rightarrow 2 \leq x \leq 3$

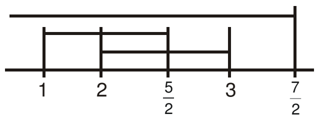
$D_2 : \ 7x - 5 - 2x^2 \geq 0 \quad \Rightarrow 2x^2 - 7x + 5 \leq 0$

$\Rightarrow (2x - 5)(x - 1) \leq 0 \quad \Rightarrow 1 \leq x \leq \frac{5}{2}$

$D_3 : \ \frac{1}{\ell n\left(\frac{7}{2}-x\right)} \qquad \Rightarrow \ell n\left(\frac{7}{2}-x\right) \leq 0$

$\Rightarrow \frac{7}{2} - x \neq 1 \qquad \Rightarrow x \neq \frac{5}{2}$

$\& \ \frac{7}{2} - x > 0 \qquad \Rightarrow x < \frac{7}{2}$



$\therefore \ x \in \left(2, \frac{5}{2}\right)$

(xix) $f(x) = \sqrt{x^2 - 5x + 4} \qquad g(x) = x + 3$

$\frac{f}{g}(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{x^2-5x+4}}{x+3}$

$x^2 - 5x + 4 \geq 0 \qquad x + 3 \neq 0$

$(x - 4)(x - 1) \geq 0 \qquad x \neq -3$

$x \geq 4 \quad x \leq 1$

$\therefore \ x \in (-\infty, -3)\ (-3, 1] \cup [4, \infty)$

Sol.2 **(i)** $y = \log_{\sqrt{5}}[(\sqrt{2}\sin x - \cos x) + 3)]$

$\Rightarrow -\sqrt{2} \leq \sin x - \cos x \ (m) \leq \sqrt{2}$

$\Rightarrow -2 \leq \sqrt{2}\, m \leq 2 \qquad \Rightarrow 1 \leq \sqrt{2}\, m + 3 \leq 5$

$\Rightarrow \log_{\sqrt{5}} \le \log_{\sqrt{5}}(\sqrt{2}m+3) \le \log_{\sqrt{5}} 5$

$\therefore\ 0 \le \log_{\sqrt{5}}(\sqrt{2}m+3) \le 2$

Range $\in$ [0, 2] ; Domain $\in$ R

(ii) $y = \dfrac{2x}{1+x^2} \Rightarrow y + yx^2 - 2x = 0$

$\Rightarrow yx^2 - 2x + y = 0$ Domain $x \in R$

$\Rightarrow D \ge 0$ $\Rightarrow 4 - 4(y^2) \ge 0$

$\Rightarrow 1 - y^2 \ge 0 \Rightarrow -1 \le y \le 1$

(iii) $f(x) = \dfrac{x^2-3x+2}{x^2+x-6} = \dfrac{(x-2)(x-1)}{(x+3)(x-2)}$

Domain $x \in R - \{-3, 2\}$

$y = \dfrac{x-1}{x+3}$; $x \neq 2$

$yx + 3y = x - 1$; when $x = 2$, $y = \dfrac{1}{5}$

$x = \dfrac{3y+1}{1-y}$ $\therefore$ Range $y \in R - \left\{\dfrac{1}{5}, 1\right\}$

(iv) $y = \dfrac{x}{1+|x|}$

If $x \ge 0 \Rightarrow y = \dfrac{x}{1+x} \Rightarrow y + yx = x$

$\Rightarrow x = \dfrac{y}{1-y} \ge 0 \Rightarrow \dfrac{y}{y-1} \le 0$

$\therefore\ 0 \le y < 1$

If $x \le 0 \Rightarrow y = \dfrac{x}{1-x} \Rightarrow x = \dfrac{y}{y+1} \le 0$

$\therefore\ y \in [-1, 0]$

$\therefore$ Domain $x \in R$, Range $\in (-1, 1)$

(v) $y = \sqrt{2-x} + \sqrt{1+x}$

$2 - x \ge 0$ $1 + x \ge 0$

$x \le 2$ $x \ge -1$

$\therefore$ Domain $x \in [-1, 2]$

$y^2 = 2 - x + 1 + x + 2\sqrt{(2-x)(1+x)}$

$= 3 + 2\sqrt{(2-x)(1+x)}$

$\therefore\ 0 \le \sqrt{(2-x)(1+x)} \le \dfrac{3}{2}$

$\Rightarrow 3 \le 3 + 2\sqrt{(2-x)(1+x)} \le 6$

$\Rightarrow \sqrt{3} \le \sqrt{3+2\sqrt{(2-x)(1+x)}} \le \sqrt{6}$

$\therefore$ Range $y \in [\sqrt{3}, \sqrt{6}]$

(vi) $f(x) = \log_{(\text{cosec } x - 1)} (2 - [\sin x] - [\sin x]^2)$

$\text{cosec } x - 1 \neq 1 \Rightarrow \text{cosec } x \neq 2$

$x \neq 2n\pi + \dfrac{\pi}{6}, \quad 2n\pi + \dfrac{5\pi}{6}$

also $\text{cosec } x \neq 1 \Rightarrow x \neq 2n\pi + \dfrac{\pi}{2}$

also $2 - t - t^2 > 0$

where $t = [\sin x]$

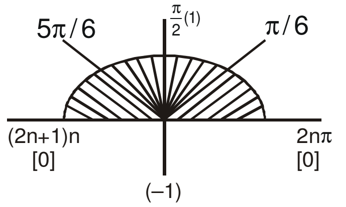
$\Rightarrow t^2 + t - 2 < 0 \Rightarrow (t + 2)(t - 1) < 0$

$\Rightarrow -2 < t < 1 \Rightarrow -2 < [\sin x] < 1$

$\Rightarrow -1 \le \sin x < 1$

but since cosec x can't be –ve hence

$0 \le \sin x < 1$



$\therefore\ x \in (2n\pi, (2n + 1)\pi) -$

$\{2n\pi + \dfrac{\pi}{2}, 2n\pi + \dfrac{\pi}{6}, 2n\pi + \dfrac{5\pi}{6}\}$ where $n \in I$

(vii) $f(x) = \dfrac{\sqrt{x+4}-3}{x-5}$

Domain : $x + 4 \ge 0 \Rightarrow x \in [-4, \infty]$ & $x \neq 5$

$\therefore\ x \in [-4, \infty) - \{5\}$

Range : $y = f(x) = \dfrac{(\sqrt{x+4}-3)(\sqrt{x+4}+3)}{(x-5)(\sqrt{x+4}+3)}$

$f(x) = \dfrac{(x-5)}{(x-5)(\sqrt{x+4}+3)}$

$\lim\limits_{x \to 5} f(x) = \lim\limits_{x \to 5} \dfrac{1}{(\sqrt{x+4}+3)} = \dfrac{1}{6}$ $(x \neq 5)$

so $x \neq 5$, $y \neq \dfrac{1}{6}$

Also $f(x) = \dfrac{1}{\sqrt{x+4}+3}$

$\Rightarrow 0 \le \sqrt{x+4} < \infty \Rightarrow 0 < \dfrac{1}{\sqrt{x+4}+3} \le \dfrac{1}{3}$

so range of y is $y \in \left(0, \dfrac{1}{6}\right) \cup \left(\dfrac{1}{6}, \dfrac{1}{3}\right]$

Sol.3 $|2x + 5| + |2x - 5| = Px + 10$

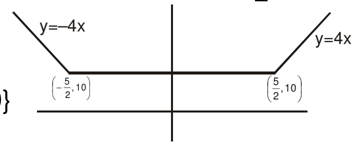
$-(2x + 5) + 5 - 2x = -4x \qquad x \le -\dfrac{5}{2}$

$2x + 5 + 5 - 2x = 10 \qquad -\dfrac{5}{2} < x < \dfrac{5}{2}$

$2x + 5 + 2x - 5 = 4x \qquad x \ge \dfrac{5}{2}$

P = slope

$P \in (-4, 4) - \{0\}$



Sol.4 $f(x) = x \quad ; \ 0 \le x \le 1$

$\quad = 1 \quad ; \ x > 1$

(i) If even then $f(-x) = f(x)$

$f(x) = -x \quad ; \ -1 \le x \le 0$

$\quad = 1 \quad ; \ x < -1$

(ii) If odd then $f(-x) = -f(x)$

$f(x) = x \quad ; \ -1 \le x \le 0$

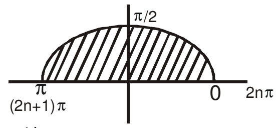
$\quad = -1 \quad ; \ x < -1$

Sol.5 $x \in [0, 1]$

(a) $f(\sin x)$

$\Rightarrow 0 \le \sin x \le 1$

$\Rightarrow 2n\pi \le x \le (2n + 1)\pi$



(b) $f(2x + 3) \ \Rightarrow 0 \le 2x + 3 \le 1$

$\Rightarrow -3 \le 2x \le -2 \ \Rightarrow -\dfrac{3}{2} \le x \le -1$

Sol.6 $f : \left[\dfrac{1}{2}, \infty\right) \to \left[\dfrac{3}{4}, \infty\right) \ ; \ f(x) = x^2 - x + 1$

$f(x) = f^{-1}(x)$

$f(x) = x \quad \Rightarrow x^2 - x + 1 = x$

$\Rightarrow x^2 - 2x + 1 = 0 \ \Rightarrow (x - 1) = 0 \ \Rightarrow x = 1$

Sol.7 **(i)** $f(x) = \ell n\,(x + \sqrt{x^2 + 1})$

$y = \ell n\,(x + \sqrt{x^2 + 1}) \Rightarrow e^y = x + \sqrt{x^2 + 1}$

$\Rightarrow e^{2y} = x^2 + x^2 + 1 + 2x\sqrt{x^2 + 1}$

$\Rightarrow e^{2y} = 2x^2 + 1 + 2x\,(e^y - x)$

$\Rightarrow e^{2y} = 2x^2 + 1 + 2xe^y - 2x^2$

$\Rightarrow 2x = \dfrac{e^{2y} - 1}{e^y} \ \Rightarrow x = \dfrac{1}{2}(e^y - e^{-y})$

$\therefore \ f^{-1}(y) = \dfrac{e^y - e^{-y}}{2}$

(ii) $f(x) = 2^{\frac{x}{x-1}}$

$\Rightarrow y = 2^{\frac{x}{x-1}} \quad \Rightarrow \log_2 y = \dfrac{x}{x-1}$

$\Rightarrow x \log_2 y - \log_2 y = x$

$\Rightarrow x\,[\log_2 y - 1] = \log_2 y$

$\Rightarrow x = \dfrac{\log y}{\log y - 1} \quad \therefore \ f^{-1}(x) = \dfrac{\log_2 x}{\log_2 x - 1}$

(iii) $y = \dfrac{10^x - 10^{-x}}{10^x + 10^{-x}} \ \Rightarrow y = \dfrac{10^{2x} - 1}{10^{2x} + 1}$

$\Rightarrow yt + y = t - 1$ where $t = 10^{2x}$

$\Rightarrow t = \dfrac{1+y}{1-y} \quad \Rightarrow 10^{2x} = \dfrac{1+y}{1-y}$

$\Rightarrow x = \dfrac{1}{2}\log\dfrac{1+y}{1-y} \quad \therefore \ f^{-1}(x) = \dfrac{1}{2}\log\dfrac{1+x}{1-x}$

Sol.8 **(i)** $10^x + 10^y = 10$

$\Rightarrow 10^y = 10 - 10^x \qquad \Rightarrow y = \log(10 - 10^x)$

Domains : $10 - 10^x > 0 \Rightarrow 10 > 10^x \Rightarrow x < 1$

(ii) $x + |y| = 2y$

If $y \ge 0$ then $\quad x + y = 2y \Rightarrow y = x$

Domain : $x \ge 0$

If $y < 0$ then $x - y = 2y \ \Rightarrow y = \dfrac{x}{3}$

Domain : $x < 0$

Sol.9 **(a)** $f(x) = \log\,(x + \sqrt{1 + x^2})$

& $f(-x) = \log\,(-x + \sqrt{1 + x^2})$

$\Rightarrow f(x) + f(-x) = \log\,(-x^2 + 1 + x^2)$

$\Rightarrow f(x) + f(-x) = 0$

$\Rightarrow f(-x) = -f(x) \qquad \therefore$ odd function

(b) $f(x) = \dfrac{x\,(a^x + 1)}{a^x - 1}$

& $f(-x) = -\dfrac{x\,(a^{-x} + 1)}{(a^{-x} - 1)} = -\dfrac{x\,(1 + a^x)}{(1 - a^x)}$

$= \dfrac{x\,(1 + a^x)}{(a^x - 1)} \qquad \therefore$ even function

(c) $f(x) = \sin x + \cos x$

& $f(-x) = -\sin x + \cos x$

$f(-x) \ne f(x) \qquad$ None

(d) $f(x) = x \sin^2 x - x^3$

& $f(-x) = -x \sin^2 x + x^3 = -f(x) \quad$ odd

(e) $f(x) = \sin x - \cos x$

& $f(-x) = -\sin x - \cos x \qquad$ None

(f) $f(x) = \dfrac{(1+2^x)^2}{2x}$

& $f(-x) = \dfrac{(1+2^{-x})^2}{2^{-x}} = \dfrac{(1+2^x)^2}{2^x}$ even

(g) $f(x) = \dfrac{x}{e^x-1} + \dfrac{x}{2} + 1$

& $f(-x) = \dfrac{-x}{e^{-x}-1} - \dfrac{x}{2} + 1$

$= \dfrac{x}{1-e^{-x}} - \dfrac{x}{2} + 1 = \dfrac{x\,e^x}{e^x-1} - \dfrac{x}{2} + 1$

$= \dfrac{x\,e^x}{e^x-1} - x + \dfrac{x}{2} + 1 = \dfrac{x\,e^x - x\,e^x + x}{e^x-1} + \dfrac{x}{2} + 1$

$f(-x) = \dfrac{x}{e^x-1} + \dfrac{x}{2} + 1$ even

(h) $f(x) = [(x+1)^2]^{1/3} + [(x-1)^2]^{1/3}$

& $f(-x) = [(-x+1)^2]^{1/3} + [(-x-1)^2]^{1/3}$

$= [(x-1)^2]^{1/3} + [(x+1)^2]^{1/3}$

$f(-x) = f(x)$ even

Sol.10 (i) $f(f(x))\,.\,(1 + f(x)) = -f(x)$

$\Rightarrow f(f(x)) = -\dfrac{f(x)}{1+f(x)} \quad \Rightarrow f(x) = \dfrac{-x}{1+x}$

$f(3) = \dfrac{-3}{1+3} = \dfrac{-3}{4}$

(ii) $f(x + f(x)) = 4f(x)$; $f(1) = 4$

$f(1 + f(1)) = 4f(1) \Rightarrow f(5) = 4.4 \Rightarrow f(5) = 16$

$f(5 + f(5)) = 4 + (5) \Rightarrow f(21) = 4.16 = 64$

(iii) $f : R^+ \to R^+$; $[f(xy)]^2 = x\,[f(y)]^2$

$x = 25$, $y = 2$

$(f(50))^2 = 25\,(f(2))^2 \Rightarrow f(50) = 5\,.f(2) = 5.6 = 30$

(iv) $f(x + y) = x + f(y)$ $f(0) = 2$

$y = 0,\ x = 2$

$f(2) = 2 + f(0) = 4$

$f(2 + 2) = 2 + f(2) = 2 + 4 = 6$

$f(2 + 4) = 2 + f(4) = 2 + 2 + 4 = 8$

$f(2 + 6) = 2 + f(6) = 2 + 2 + 2 + 4 = 10$

$f(100) = 102$

(v) $f(3) = 1$

$f(3x) = x + f(3x - 3)$

$x = 1$ $f(3) = 1 + f(0) \Rightarrow f(0) = 0$

$x = 2$ $f(6) = 2 + f(3) = 2 + 1$

$x = 3$ $f(9) = 3 + f(6) = 3 + 2 + 1$

$x = 4$ $f(12) = 4 + f(a) = 4 + 3 + 2 + 1$

$f(3 \times 100) = 100 + 99 + 98 + \ldots\ldots. + 1$

$= 5050$

Sol.11 $f(x + a) = \dfrac{1}{2} + \sqrt{f - f^2}$

replace $x \to x + a$ [where $f' = f(x + a)$]

$f(x + 2a) = \dfrac{1}{2} + \sqrt{f' - f'^2} = \dfrac{1}{2} + \sqrt{f'(1-f')}$

$= \dfrac{1}{2} + \sqrt{\left(\dfrac{1}{2} + \sqrt{f-f^2}\right)\left(\dfrac{1}{2} - \sqrt{f-f^2}\right)}$

$= \dfrac{1}{2} + \sqrt{\dfrac{1}{4} - f + f^2}$

$f(x + 2a) = \dfrac{1}{2} + \left|f - \dfrac{1}{2}\right| = \dfrac{1}{2} + f - \dfrac{1}{2}$

$f(x + 2a) = f(x)$

Sol.12 $f(1) = 2$; $f(2) = 8$

$f(x + y) - k\,xy = f(x) + 2y^2$ Put $y = 1$, $k = 4$

$f(1 + y) = 4y + f(1) + 2y^2 = 4y + 2 + 2y^2$

$= 2\,(y^2 + 2y + 1) = 2(y + 1)^2$

$f(x) = 2x^2$; $f(x + y) = 2(x + y)^2$; $f\left(\dfrac{1}{x+y}\right) = \dfrac{2}{(x+y)^2}$

$f(x + y)\, f\left(\dfrac{1}{x+y}\right) = 4 = k$

Sol.13 $f(x) = \begin{cases} e^{-\sqrt{|\ell n\{x\}|}} - \{x\}^{\sqrt{\frac{1}{|\ell n\{x\}|}}} & ;\ \text{where ever it exists} \\ \{x\} & ;\ \text{otherwise, then} \end{cases}$

For first definition of f(x)

as $0 \le \{x\} < 1$ But $\{x\} \ne 0 \Rightarrow x \notin I$

$f(x) = \begin{cases} e^{-\sqrt{|\ell n\{x\}|}} - \{x\}^{\sqrt{\frac{1}{|\ell n\{x\}|}}} & ;\ x \notin I \\ \{x\} & ;\ x \in I \end{cases}$

$f(x) = e^{-\sqrt{-\ell n\{x\}}} - \{x\}^{\sqrt{\frac{-1}{\ell n\{x\}}}}$

$= e^{-t} - e^{\frac{1}{t}\ell n\{x\}}$ (where $t = \sqrt{-\ell n\{x\}}$)

$= e^{-t} - e^{-t} = 0$

$f(x) = 0$

$f(x) = 0$ is only the function which is odd as well as even.

Sol.14 $f(x) = \dfrac{9^x}{9^x + 3}$

$$f\left(\frac{1}{2006}\right) = \frac{9^{1/2006}}{9^{1/2006} + 3}$$

$$f\left(\frac{2005}{2006}\right) = \frac{9^{\frac{2005}{2006}}}{9^{\frac{2005}{2006}} + 3} = \frac{9.9^{-1/2006}}{9.9^{-1/2006} + 3}$$

$$= \frac{9}{9 + 3.9^{1/2006}} = \frac{3}{3 + 9^{1/2006}}$$

$$\left.\begin{array}{l} f\left(\frac{1}{2006}\right) + f\left(\frac{2005}{2006}\right) = 1 \\ \vdots \\ f\left(\frac{2}{2006}\right) + f\left(\frac{2004}{2006}\right) = 1 \\ \vdots \\ f\left(\frac{1002}{2006}\right) + f\left(\frac{1004}{2006}\right) = 1 \end{array}\right\} \Rightarrow 1002$$

$$f\left(\frac{1003}{2006}\right) = \frac{3}{3+3} = \frac{1}{2}$$

sum = 1002 + 0.5 = 1002 . 5

Sol.15 $\left[\dfrac{3}{x}\right] + \left[\dfrac{4}{x}\right] = 5$; $x > 0$

Case-I : $\left[\dfrac{3}{x}\right] = 1$ & $\left[\dfrac{4}{x}\right] = 4$

$1 \le \dfrac{3}{x} < 2$ $\quad$ $4 \le \dfrac{4}{x} < 5$

$\dfrac{3}{2} < x \le 3$ $\quad$ $\dfrac{4}{5} < x \le 1$

$x \in \phi$

Case–II : $\left[\dfrac{3}{x}\right] = 2$ & $\left[\dfrac{4}{x}\right] = 3$

$2 \le \dfrac{3}{x} < 3$ $\quad$ $3 \le \dfrac{4}{x} < 4$

$1 < x \le \dfrac{3}{2}$ $\quad$ $1 < x \le \dfrac{4}{3}$

$x \in \left(1, \dfrac{4}{3}\right]$

a = 1 ; b = 4 ; c = 3

a + b + c + abc = 1 + 4 + 3 + 12 = 20

Sol.16 $f : R \to R$; $f\left(\dfrac{1-x}{1+x}\right) = x$

Put $\dfrac{1-x}{1+x} = t \Rightarrow x = \dfrac{1-t}{1+t}$

$\therefore f(t) = \dfrac{1-t}{1+t} \Rightarrow f(x) = \dfrac{1-x}{1+x}$

(a) $f[f(x)] = f\left(\dfrac{1-x}{1+x}\right) = \dfrac{1 - \left(\frac{1-x}{1+x}\right)}{1 + \left(\frac{1-x}{1+x}\right)} = x$

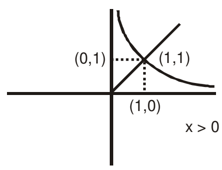
(b) $f\left(\dfrac{1}{n}\right) = \dfrac{1 - 1/x}{1 + 1/x} = \dfrac{x-1}{x+1} = -f(x)$

(c) $f(-x-2) = \dfrac{1-(-x-2)}{1+(-x-2)} = \dfrac{1+x+2}{1-x-2}$

$$= \frac{1+2x-x+2}{-(1+x)} = -\left(\frac{1-x}{1+x}\right) - 2 = -f(x) - 2$$

Sol.17 $f(x) = \max\left(x, \dfrac{1}{x}\right)$ $\quad$ $x > 0$

$= \max(a, b)$



$g(x) = f(x).\, f\left(\dfrac{1}{x}\right)$

$f(x) = \dfrac{1}{x}$ $\quad$; $0 < x \le 1$

$= x$ $\quad$; $x > 1$

$f\left(\dfrac{1}{x}\right) = x$ $\quad$; $0 < \dfrac{1}{x} \le 1 \Rightarrow x \ge 1$

$= \dfrac{1}{x}$ $\quad$; $\dfrac{1}{x} > 1 \Rightarrow x < 1$

$g(x) = f(x) \,.\, f\left(\dfrac{1}{x}\right) = x^2$ $\quad$; $x > 1$

$= \dfrac{1}{x^2}$ $\quad$; $0 < x < 1$

Sol.18 True False False

(i) If $f(x) = 1$ $f(y) \neq 1$ $f(z) \neq 2$

$f(x) = 1$ $f(y) = 1$ $f(z) = 2$

But function is one-one so not possible

False True False

(ii) If $f(x) = 1$ $f(y) \neq 1$ $f(z) \neq 2$

$f(x) = 3$ $f(y) = 1$ $f(z) = 2$

$f^{-1}(1) = y$

False False True

(iii) If $f(x) = 1$ $f(y) \neq 1$ $f(z) \neq 2$

$f(x) = 2$ $f(y) = 1$ $f(z) = 3$

$f^{-1}(1) = y$

(d) $P(x) = x^6 + ax^5 + bx^4 + cx^3 + dx^2 + ex + f$

possible p(x) will be

$P(x) = (x-1)(x-2)(x-3)(x-4)(x-5)(x-6) + x$

$P(7) = 6! + 7 = 720 + 7 = 727$

(e) $f(x) = a \sin x + b(x)^{1/3} + 4$

$f(\log_{10}(\log_3 10))$

$= a\sin(\log_{10}(\log_3 10)) + b(\log_{10}(\log_3 10))^{1/3} + 4 = 5$

$= a\sin(\log_{10}(\log_{10} 3)^{-1}) + b(\log_{10}(\log_{10} 3)^{-1})^{1/3} + 4 = 5$

$= -a\sin(\log_{10}(\log_{10} 3)) - b(\log_{10}\log_{10} 3) = 1$

$= -(M) = 1 \Rightarrow M = -1$

$f(\log_{10}\log_{10} 3) = a\sin(\log_{10}\log_{10} 3) + b(\log_{10}\log_{10} 3) + 4$

$= -1 + 4 = 3$

Sol.19 (a) $f(1) + f(2) = 4f(2) \Rightarrow f(2) = \dfrac{f(1)}{3}$

$f(1) + f(2) + f(3) = 3 \Rightarrow f(3) = \dfrac{f(1)}{6}$

$f(1) + f(2) + f(3) + f(4) = 16 \Rightarrow f(4) = \dfrac{f(1)}{10}$

$\Rightarrow \quad f(n) = \dfrac{2f(1)}{n(n+1)}$

$f(2004) = \dfrac{2f(1)}{2004 \times 2005}$

$= \dfrac{1}{1002}$ (as $f(1) = 2005$)

(b) Given $a - b = 2$

$$\sqrt{x^2+ax} - \sqrt{x^2+bx} < L$$

$$\frac{(a-b)x}{\sqrt{x^2+ax}+\sqrt{x^2+bx}} < L$$

$$\frac{2x}{\sqrt{x^2+ax}+\sqrt{x^2+bx}} < L$$

$$\frac{2}{\sqrt{1+\frac{a}{x}}+\sqrt{1+\frac{b}{x}}} < L$$

$L > 1$ so min. value of $L = 1$

(c) $f(f(x)) = (x^2 + kx)^2 + k(x^2 + kx) = 0$

$(x^2 + kx)\ \{x^2 + kx + k\} = 0$

for same solution $x^2 + kx + k = 0$ must not have any real roots

$D < 0$

$k^2 - 4k < 0$

$k \in (0, 4)$

check for end values of k {k = 0 is possible}

so $k \in [0, 4)$

Sol.20 $\left[x - \frac{1}{2}\right]\left[x + \frac{1}{2}\right]$ is prime So

Case-I : $1 \le x - \frac{1}{2} < 2$ & $2 \le x + \frac{1}{2} < 3$

$\frac{3}{2} \le x < \frac{5}{2}$ $\qquad \frac{3}{2} \le x < \frac{5}{2}$

so $x \in \left(\frac{3}{2}, \frac{5}{2}\right]$

Case-II : $-2 \le x - \frac{1}{2} < -1$ & $-1 \le x + \frac{1}{2} < 0$

$-\frac{3}{2} \le x < -\frac{1}{2}$ $\qquad -\frac{3}{2} \le x < -\frac{1}{2}$

$x \in \left[-\frac{3}{2}, \frac{-1}{2}\right)$

Final $x \in \left[-\frac{3}{2}, \frac{-1}{2}\right) \cup \left[\frac{3}{2}, \frac{5}{2}\right)$

$x_1 = \frac{3}{2}$; $x_2 = \frac{-1}{2}$; $x_3 = \frac{3}{2}$; $x_4 = \frac{5}{2}$

$x_1^2 + x_2^2 + x_3^2 + x_4^2 = \dfrac{9+1+9+25}{4} = 11$

Sol.21 $P(x) = (x-1)Q_1 + 1 \quad \Rightarrow \quad P(1) = 1$

$P(x) = (x-4)Q_2 + 10 \quad \Rightarrow \quad P(4) = 10$

$P(x) = (x-1)(x-4)Q_3 + (ax + b)$

$P(1) = 1 \quad \Rightarrow a + b = 1$

$P(4) = 10 \quad \Rightarrow 4a + b = 10$

$a = 3$; $b = -2$

$r(x) = 3x - 2 \Rightarrow r(2006) = 6016$

Sol.22 $f(x + T) = \dfrac{f(x)-5}{f(x)-3}$ ....(i)

Replace x by x + T

$f(x+2T) = \dfrac{f(x+T)-5}{f(x+T)-3} = \dfrac{2f(x)-5}{f(x)-2}$ ....(ii) (from (i))

Replace x by x + 2T

$f(x + 4T) = \dfrac{2f(x+2T)-5}{f(x+2T)-2} = f(x)$ (from (ii))

f(x) is periodic with period 4T

Sol.23 $f(x) = x^{135} + x^{125} - x^{115} + x^5 + 1$

$x^{135} + x^{125} - x^{115} + x^5 + 1 = (x^3 - x)\,Q(x) + (ax^2 + bx + c)$

put $x = 0 \Rightarrow c = 1$

put $x = 1 \Rightarrow a + b = 2$

put $x = -1 \Rightarrow a - b = -2$

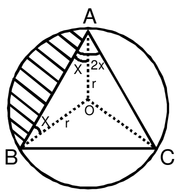
$a = 0,\ b = 2$

$g(x) = 2x + 1 \Rightarrow g(10) = 21$

Sol.24 Area of shaded region = area of arc OAB – area of ΔOAB

$\dfrac{\pi r^2}{3} = \dfrac{1}{2} r^2 (\pi - 2x) - \dfrac{1}{2} r^2 \sin 2x$

$\pi - 2x - \sin 2x = \dfrac{2\pi}{3}$



$f(2x) = 2x + \sin 2x - \dfrac{\pi}{3}$

$f(x) = x + \sin x - \dfrac{\pi}{3}$

EXERCISE – V

HINTS & SOLUTIONS

Sol.1 B

$f : [1, \infty) \to [1, \infty)$; $y = f(x) \Rightarrow x = f^{-1}(y)$

$y = 2^{x(x-1)} \Rightarrow \log_2 y = x(x-1)$

$\Rightarrow x^2 - x - \log_2 y = 0 \Rightarrow x = \dfrac{1 \pm \sqrt{1 + 4\log_2 y}}{2}$

x should be positive so $x = \dfrac{1 + \sqrt{1 + 4\log_2 y}}{2}$

$\therefore f^{-1}(y) = \dfrac{1 + \sqrt{1 + 4\log_2 y}}{2}$

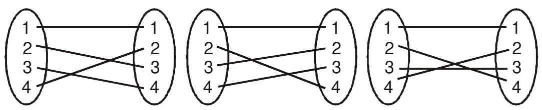
$\Rightarrow f^{-1}(x) = \dfrac{1 + \sqrt{1 + 4\log_2 x}}{2}$

Sol.2 D

$2^y + 2^x = 2 \Rightarrow 2^y = 2 - 2^x \Rightarrow y = \log(2 - 2^x)$

$\therefore 2 - 2^x > 0 \Rightarrow 2^x < 2 \Rightarrow x < 1$

Sol.3 $x = \{1, 2, 3, 4\}$; $f : x \to x$



Sol.4 (a) B

$$g(x) = 1 + x - [x] \ ; \ f(x) = \begin{cases} -1 & x < 0 \\ 0 & x = 0 \\ 1 & x > 0 \end{cases}$$

we know that $x - [x] = \{x\}$

$\therefore \quad g(x) = 1 + \{x\} > 1 \ \forall \ x \in R$

or $f[g(x)] = 1$ from definition of $f(x) = 1, x > 0$

(b) A

$f : [1, \infty) \to [2, \infty)$; $f(x) = y \Rightarrow x = f^{-1}(y)$

$y = x + \dfrac{1}{x} \Rightarrow x^2 - xy + 1 = 0 \Rightarrow x = \dfrac{y \pm \sqrt{y^2 - 4}}{2}$

x should be greater than 1 so

$x = \dfrac{y + \sqrt{y^2 - 4}}{2} \Rightarrow f^{-1}(y) = \dfrac{y + \sqrt{y^2 - 4}}{2}$

$\therefore f^{-1}(x) = \dfrac{x + \sqrt{x^2 - 4}}{2}$

(c) D

$f(x) = \dfrac{\log_2(x+3)}{x^2 + 3x + 2} = \dfrac{\log_2(x+3)}{(x+1)(x+2)}$

$x + 3 > 0 \Rightarrow x > -3$ & $x \neq -1, -2$

(d) A

$E \to F$; Number of functions $= 2^4 = 16$

Each element of E is connect with element 1 of F so 2 is left.

when each element of E is connect with Element 2 of F so 1 is left

so function is onto in two situation

so number of onto functions = 16 – 2 = 14

(e) D

$f(x) = \dfrac{\alpha x}{x+1}$

$f(f(x)) = \dfrac{\alpha\left(\dfrac{\alpha x}{x+1}\right)}{\dfrac{\alpha x}{x+1} + 1} = x \Rightarrow \dfrac{\alpha^2 x}{\alpha x + x + 1} = x$

$\Rightarrow \alpha^2 = (\alpha + 1)x + 1$

$\therefore \alpha^2 = 1 \Rightarrow \alpha = \pm 1, \alpha + 1 = 0 \Rightarrow \alpha = -1$

so $\alpha = -1$

Sol.5 (a) D

clearly g(x) will be inverse of f(x)

$f(x) = y \Rightarrow x = f^{-1}(y)$

$y = (x+1)^2 \Rightarrow x + 1 = \pm\sqrt{y} \Rightarrow x = -1 \pm \sqrt{y}$

$\because x \geq -1 \Rightarrow x = -1 + \sqrt{y}$

$\Rightarrow f^{-1}(y) = -1 + \sqrt{y} \ \therefore f^{-1}(x) = -1 + \sqrt{x}$

(b) A

$f(x) = 2x + \sin x$

$f'(x) = 2 + \cos x > 0 \ \forall \ x \in R$

Also $f(x) \to \infty$ as $x \to \infty$ & $f(x) \to -\infty$ as $x \to -\infty$

Thus f(x) is one-one and onto

Sol.6 **(a)** **A**

$$f'(x) = \frac{1+x-x}{(1+x^2)^2} = \frac{1}{(1+x^2)^2}$$

$\Rightarrow$ f is increasing $\Rightarrow$ f is one-one

Range of f = [0, 1) $\Rightarrow$ f is not onto

(b) **D**

$$y = \frac{x^2+x+2}{x^2+x+1} \Rightarrow x^2 + x + 2 = x^2y + xy + y$$

$\Rightarrow x^2(1-y) + x(1-y) + 2 - y = 0$ ($\because y \neq 1$)

$D \geq 0 \Rightarrow (1-y)^2 - 4(1-y)(2-y) \geq 0$

$\Rightarrow 1 + y^2 - 2y - 4(2 - 3y + y^2) \geq 0$

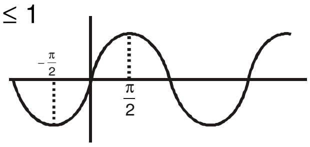
$\Rightarrow 3y^2 - 10y + 7 \leq 0$

$$\Rightarrow 1 \leq y \leq \frac{7}{3} \quad \therefore \quad 1 < y \leq \frac{7}{3}$$

Sol.7 $g(f(x)) = (\sin x + \cos x)^2 - 1 = \sin 2x$

$-1 \leq \sin 2x \leq 1$

$$-\frac{\pi}{2} \leq 2x \leq \frac{\pi}{2}$$



$$-\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$$

Sol.8 **A**

$$\text{Let } h(x) = f(x) - g(x) = \begin{cases} x \ ; \ x \in \text{irrational} \\ -x \ ; \ x \in \text{rational} \end{cases}$$

$\Rightarrow$ the function h(x) is one-one and onto.

Sol.9 **D**

Each element in either (A) or (B) or neither.

$\therefore$ Total ways = $3^4 = 81$

A = B iff A = B = ϕ (1 case)

otherwise A & B are interchangeable

$\therefore$ n = 1 + (81 – 1) / 2 = 41

10. **A**

fogogof(x) = $\sin^2(\sin x^2)$

gogof(x) = $\sin(\sin x^2)$

on solving, $\sin(\sin x^2) = 0$ or1

$\Rightarrow$ $x = \pm\sqrt{n\pi}$; $n \in \{0, 1, 2 ......\}$

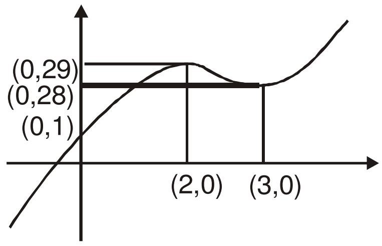
11. **B**

$f(X) = 2X^3 - 15X^2 + 36X + 1$

$f'(X) = 6X^2 - 30X + 36$

$= 6(X^2 - 5X + 6)$

$= 6(x-3)(x-2)$



f(0) = 1

f(2) = 2 × 8 – 15 × 4 + 36 × 2 +1 = 29

f(3) = 2 × 27 – 15 × 9 + 36 × 3 + 1 = 28

onto but not one one so 'B'.

12. **A,B**

$$f(\cos 4\theta) = \frac{2\cos^2\theta}{2\cos^2\theta - 1} = \frac{1+\cos 2\theta}{\cos 2\theta}$$

$$\Rightarrow 2\cos^2 2\theta - 1 = \frac{1}{3}$$

$$\Rightarrow \cos^2 2\theta = \frac{2}{3} \Rightarrow \cos 2\theta = \pm\sqrt{\frac{2}{3}}$$

$$f\left(\frac{1}{3}\right) = 1 \pm \sqrt{\frac{3}{2}}$$

Answer Ex–I SINGLE CORRECT (OBJECTIVE QUESTIONS)

1. D	**2.** D	**3.** B	**4.** A	**5.** A	**6.** A	**7.** D
8. B	**9.** B	**10.** D	**11.** D	**12.** A	**13.** C	**14.** B
15. D	**16.** C	**17.** C	**18.** B	**19.** A	**20.** A	**21.** B
22. C	**23.** B	**24.** C	**25.** B	**26.** D	**27.** A	**28.** A
29. C	**30.** C	**31.** D	**32.** D	**33.** D	**34.** B	**35.** C
36. D	**37.** C	**38.** B	**39.** B	**40.** A	**41.** D	**42.** D
43. C	**44.** B	**45.** D	**46.** D	**47.** B	**48.** D	**49.** B
50. B	**51.** C	**52.** D	**53.** C	**54.** D	**55.** A	**56.** C
57. A	**58.** A	**59.** C	**60.** C	**61.** B		

Answer Ex–II MULTIPLE CORRECT (OBJECTIVE QUESTIONS)

1. A,B	**2.** B,C,D	**3.** A,C	**4.** A,C	**5.** A,B,C	**6.** A,B,C	**7.** B,C,D
8. B,C,D	**9.** A,D	**10.** A,B,C,D				

Answer Ex–III SUBJECTIVE QUESTIONS

1. **(i)** $R - \{-1, 1\}$ **(ii)** $(0, \infty)$ **(iii)** R **(iv)** $[-2, 0) \cup (0, 1)$ **(v)** $\left(\frac{1}{2}, 1\right) \cup \left(1, \frac{3}{2}\right)$

(vi) $[0, 1]$ **(vii)** $[-1, 1]$ **(viii)** R **(ix)** ϕ **(x)** $\bigcup_{n \in I}\left[n\pi, n\pi + \frac{\pi}{4}\right]$

(xi) $R - \{2n\pi\}, n \in I$ **(xii)** $(0, 1) \cup [4, 5)$ **(xiii)** $(2, 3)$

2. **(i)** $[0, \infty)$ **(ii)** $\left[-\frac{1}{2}, \frac{1}{2}\right]$ **(iii)** $[0, 4]$ **(iv)** $\{-1, 1\}$ **(v)** $[0, 10]$ **(vi)** $(0, 1]$

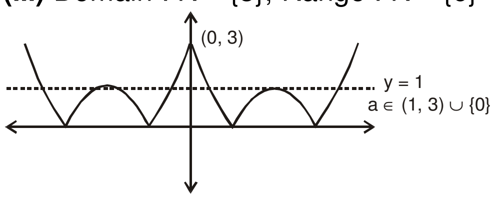
(vii) $(-\infty, \frac{49}{20}]$ **(viii)** $[-4, 3]$ **(ix)** $[-1, 1]$ **(x)** $(-\infty, 1]$ **(xi)** R^+ **(xii)** $\left[\frac{1}{3}, 1\right]$

(xiii) $\left(-\infty, -\frac{1}{4}\right] \cup \left[-\frac{1}{20}, \infty\right)$ **(xiv)** $\left[\frac{1}{3}, 3\right]$

(xv) $\left[0, \frac{3}{\sqrt{2}}\right]$ **(xvi)** $[4, \infty)$ **(xvii)** $[-11, 16]$ **(xviii)** $\left[\frac{3}{4}, 1\right]$

3. **(i)** Domain : R, Range : $\frac{1}{\sqrt{7}} \le y \le 1$ **(ii)** Domain : $N \cup \{0\}$, Range : $\{n! : n = 0, 1, 2, ...\}$

(iii) Domain : $R - \{3\}$, Range : $R - \{6\}$ **(iv)** Domain : R, Range : $\{1\}$

5. 

6. $\left\{\frac{3}{2}\right\}$ **7.** $g(x) = \begin{cases} -x & , \ -2 \le x < 0 \\ 0 & , \ 0 \le x \le 1 \\ 2(x-1) & , \ 1 < x \le 2 \end{cases}$

8. **(i)** No. **(ii)** Yes **(iii)** No **(iv)** No

9. **(i)** many–one **(ii)** many–one **(iii)** one–one **(iv)** many–one **(v)** one–one **(vi)** many–one

10. **(i)** into **(ii)** onto **(iii)** into **(iv)** onto

11. **(i)** bijective (injective as well as surjective) **(ii)** neither injective nor surjective
(iii) neither surjective nor injective **(iv)** surjective but not injective

12. **(i)** No **(ii)** Yes **(iii)** yes **(iv)** No

13. **(i)** fog = x, x > 0 ; gof = x, $x \in R$ **(ii)** |sin x|, sin |x|

(iii) $\sin^{-1}(x^2)$, $(\sin^{-1} x^2)$ **(iv)** $\dfrac{3x^2-4x+2}{(1-x)^2}, \dfrac{x^2+2}{x^2+1}$

14. $f(g(x)) = \begin{cases} 2-2x+x^2 & 0 \le x \le 1 \\ 2-x & -1 \le x < 0 \end{cases}$ **15.** $(fof)(x) = \begin{cases} 2+x, & 0 \le x \le 1 \\ 2-x, & 1 < x \le 2 \\ 4-x, & 2 < x \le 3 \end{cases}$

16. Domain : [1, 2] ; Range : [$\ln 2$, $\ln 4$) **17.** –3 **18.** 2046

19. **(i)** $f(x) = \begin{cases} x^2 - \sin x & -1 < x \le 0 \\ -x + e^x & x \le -1 \end{cases}$ **(ii)** $f(x) = \begin{cases} -x^2 + \sin x & -1 < x \le 0 \\ x - e^x & x \le -1 \end{cases}$

20. **(i)** neither even nor odd **(ii)** even **(iii)** odd **(iv)** even **(v)** odd

21. **(i)** 2π **(ii)** 2π **(iii)** 24 **(iv)** 70π
(v) $\dfrac{2\pi}{3}$ **(vi)** 2π **(vii)** $\pi/2$ **(viii)** 2π

22. **(i)** π **(ii)** $\dfrac{2\pi}{3}$ **(iii)** 2 **(iv)** 2π **(v)** 2π **(vi)** $2^n \pi$ **(vii)** π

23. **(i)** 2p **(ii)** 8 **24.** $f^{-1} : R \to R$, $f^{-1}(x) = \dfrac{1}{2} \ln\left(x + \sqrt{x^2+1}\right)$

25. B = [0, 4]; $f^{-1}(x) = \dfrac{1}{2}\left(\sin^{-1}\left(\dfrac{x-2}{2}\right) - \dfrac{\pi}{6}\right)$ **26.** $f^{-1}(x) = x + (-1)^{x-1}$, $x \in N$

Answer Ex–IV ADVANCED SUBJECTIVE QUESTIONS

1. **(i)** $\left[-\dfrac{5\pi}{4}, \dfrac{-3\pi}{4}\right] \cup \left[-\dfrac{\pi}{4}, \dfrac{\pi}{4}\right] \cup \left[\dfrac{3\pi}{4}, \dfrac{5\pi}{4}\right]$ **(ii)** $\left(-4, -\dfrac{1}{2}\right) \cup (2, \infty)$ **(iii)** $(-\infty, -3]$

(iv) $(-\infty, -1) \cup [0, \infty)$ **(v)** $(3 - 2\pi < x < 3 - \pi)$ U $(3 < x \le 4)$

(vi) $\left(0, \dfrac{1}{100}\right) \cup \left(\dfrac{1}{100}, \dfrac{1}{\sqrt{10}}\right)$ **(vii)** $(-1 < x < -1/2)$ U $(x > 1)$

(viii) $\left[\dfrac{1-\sqrt{5}}{2}, 0\right) \cup \left[\dfrac{1+\sqrt{5}}{2}, \infty\right)$ **(ix)** (–3, –1] U {0} U [1,3)

(x) $\{4\} \cup [5, \infty)$ **(xi)** (0 , 1/4) U (3/4 , 1) U $\{x : x \in N, x \ge 2\}$

(xii) $\left(-\dfrac{1}{6}, \dfrac{\pi}{3}\right] \cup \left[\dfrac{5\pi}{3}, 6\right)$ **(xiii)** $[-3, -2) \cup [3, 4)$ **(xiv)** ϕ

(xv) $2K\pi < x < (2K+1)\pi$ but $x \neq 1$ where K is non–negative integer

(xvi) $\{x \mid 1000 \le x < 10000\}$ **(xvii)** $(-2, -1) \cup (-1, 0) \cup (1, 2)$

(xviii) $(1, 2) \cup \left(2, \frac{5}{2}\right)$ **(xix)** $(-\infty, -3) \cup (-3, 1] \cup [4, \infty)$

2. **(i)** D : $x \in R$ R : $[0, 2]$ **(ii)** D = R ; range $[-1, 1]$

(iii) D : $\{x \mid x \in R \; ; \; x \neq -3 \; ; \; x \neq 2\}$; R : $\{f(x) \mid f(x) \in R, \; f(x) \neq 1/5 \; ; \; f(x) \neq 1\}$

(iv) D : $x \in R$; range $\in (-1, 1)$ **(v)** D : $x \in [-1, 2]$; range $\in [\sqrt{3}, \sqrt{6}]$

(vi) D : $x \in (2n\pi, (2n+1)\pi) - \{2n\pi + \frac{\pi}{6}, 2n\pi + \frac{\pi}{2}, 2n\pi + \frac{5\pi}{6}, n \in I\}$ and

R : $\log_a 2$; $a \in (0, \infty) - \{1\} \Rightarrow$ Range is $(-\infty, \infty) - \{0\}$

(vii) D : $[-4, \infty) - \{5\}$; R : $\left(0, \frac{1}{6}\right) \cup \left(\frac{1}{6}, \frac{1}{3}\right]$

3. $p \in (-4, 4) - \{0\}$

4. **(i)** $f(x) = 1$ for $x < -1$ & $-x$ for $-1 \le x \le 0$; **(ii)** $f(x) = -1$ for $x < -1$ and x for $-1 \le x \le 0$.

5. **(a)** $2K\pi \le x \le 2K\pi + \pi/2$ where $K \in I$ **(b)** $[-3/2, -1]$

6. $x = 1$ **7.** **(i)** $\frac{e^x - e^{-x}}{2}$ **(ii)** $\frac{\log_2 x}{\log_2 x - 1}$ **(iii)** $\frac{1}{2} \log \frac{1+x}{1-x}$

8. **(i)** $y = \log(10 - 10^x)$, $-\infty < x < 1$ **(ii)** $y = x/3$ when $-\infty < x < 0$ & $y = x$ when $0 \le x < +\infty$

9. **(a)** odd **(b)** even **(c)** neither odd nor even **(d)** odd

(e) neither odd nor even **(f)** even **(g)** even **(h)** even

10. **(i)** –3/4, **(ii)** 64 **(iii)** 30 **(iv)** 102 **(v)** 5050

12. $f(x) = 2x^2$ **14.** 1002.5 **15.** 20 **17.** $g(x) = \begin{cases} \frac{1}{x^2} \text{ if } 0 < x \le 1 \\ x^2 \text{ if } x > 1 \end{cases}$

18. $f^{-1}(1) = y$ **19.** **(a)** $\frac{1}{1002}$, **(b)** 1, **(c)** [0, 4), **(d)** 727, **(e)** 3

20. 11 **21.** 6016 **23.** 21 **24.** $f(x) = \sin x + x - \frac{\pi}{3}$

Answer Ex–V JEE PROBLEMS

1. B **2.** **(a)** C, **(b)** D, **(c)** B, **(d)** C

3. {(1,1), (2, 3), (3, 4), (4, 2)} ; {(1,1), (2, 4), (3, 2), (4, 3)} and {(1,1), (2, 4), (3, 3), (4, 2)}

4. **(a)** B, **(b)** A, **(c)** D, **(d)** D, **(e)** A, **(f)** D **5.** **(a)** D, **(b)** A **6.** **(a)** A, **(b)** D, **(c)** A

7. C **8.** **(a)** A, **(b)** C, **(c)** $t \in \left[-\frac{\pi}{2}, \frac{\pi}{10}\right] \cup \left[\frac{3\pi}{10}, \frac{\pi}{2}\right]$ **9.** **(a)** D, **(b)** B, **(c)** A **10.** D

11. B **12.** A,B